Control Approaches for Walking and Running

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HELMHOLTZ | ASSOCIATION

DLR

Overview

1) Humanoid robot TORO

2) Walking Control

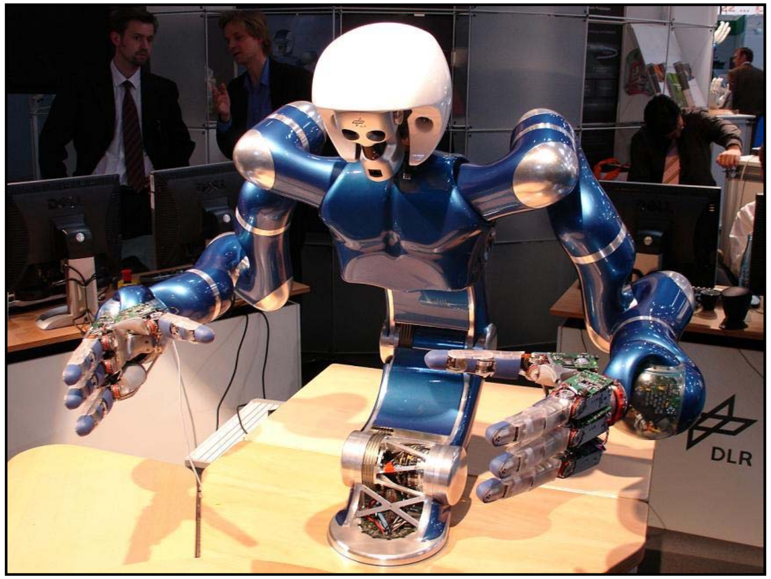
   - ✓ Capture Point
   - ✓ Divergent Component of Motion (3D)

3) Running

Humanoid Robots at DLR

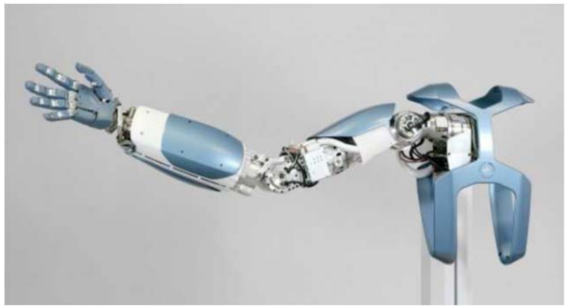
Joint torque sensing & control

Bimanual (Humanoid) Manipulation



Legged Humanoid

Space Qualified Joint Technology



- Compliant actuation
- Antagonistic actuation for fingers
- Variable stiffness actuation in arm
- Robustness to shocks and impacts

Anthropomorphic Hand-Arm System

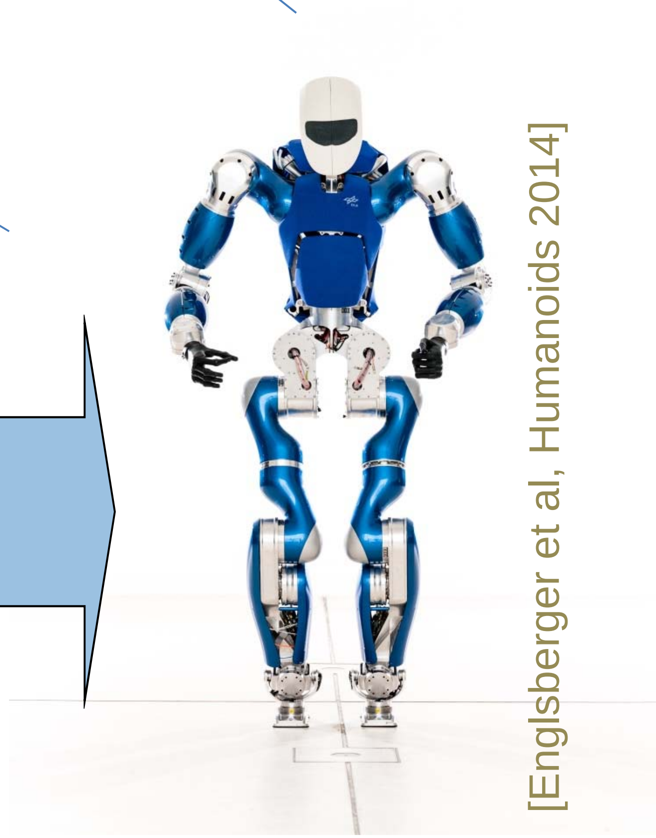
Bipedal Walking Robots at DLR

- Joint torque sensing & control
- Small foot size: 19 x 9,5 cm
- IMU in head & trunk
- FTS in feet for position based control
- Sensorized head (stereo vision & kinect)
- Simple prosthetic hands (iLIMB)

[Ott et al, Humanoids 2010]

DLR-Biped
(2010-2012)

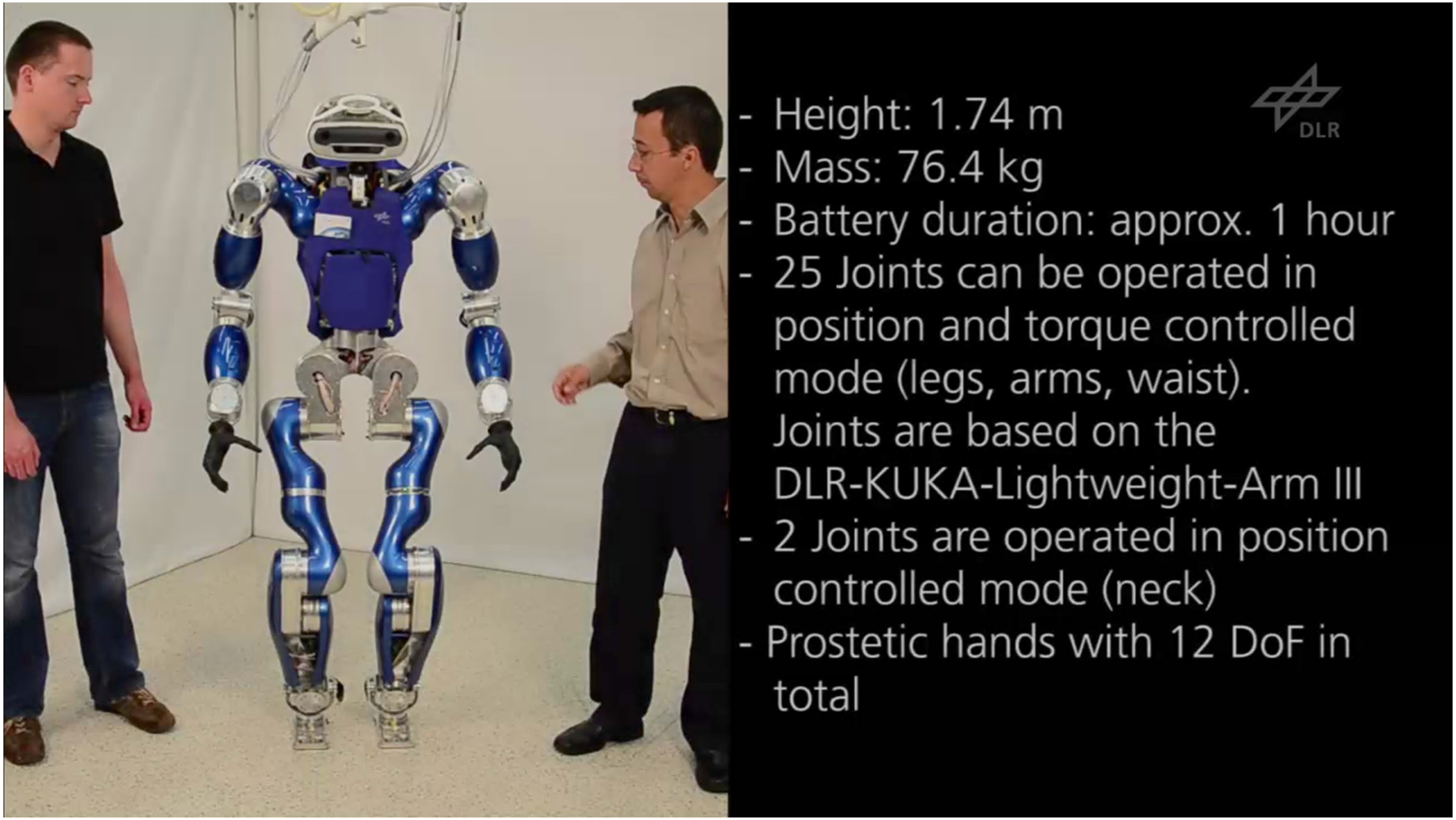
TORO, preliminary version
(2012)

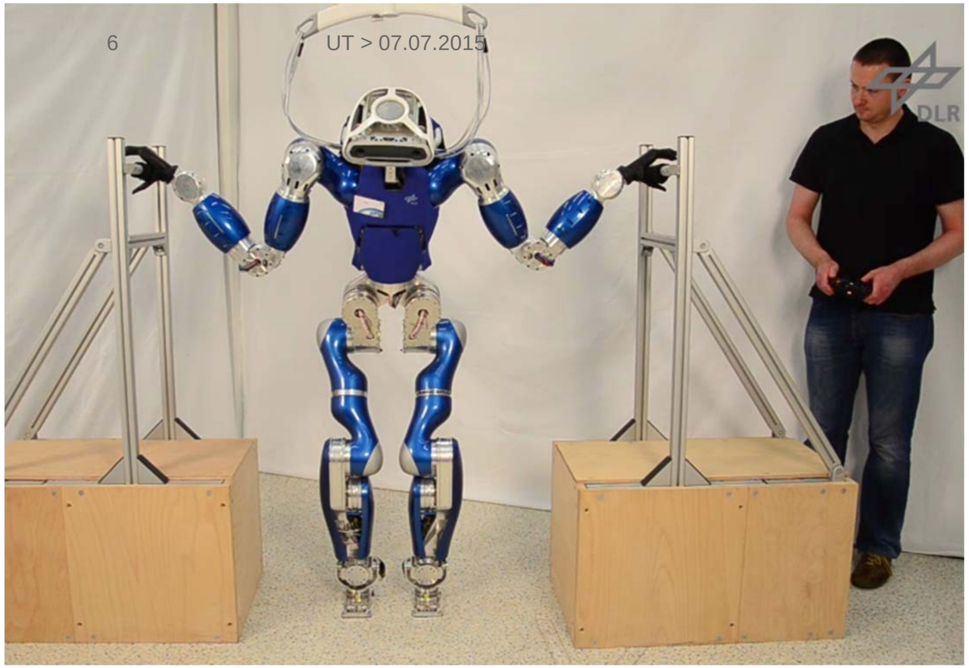


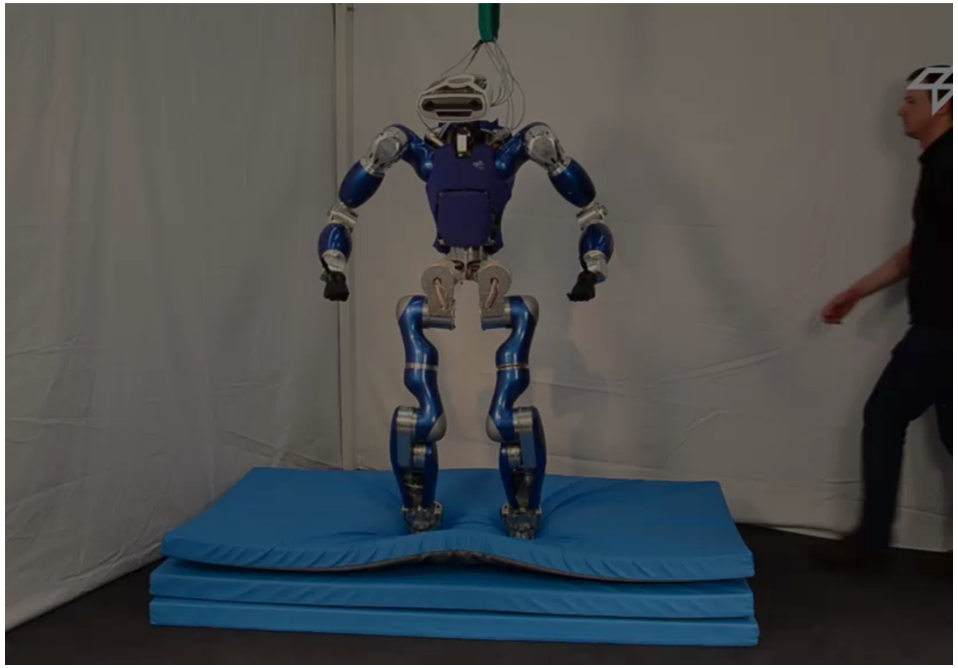
TORO (2013)
TOrque controlled
humanoid RObot

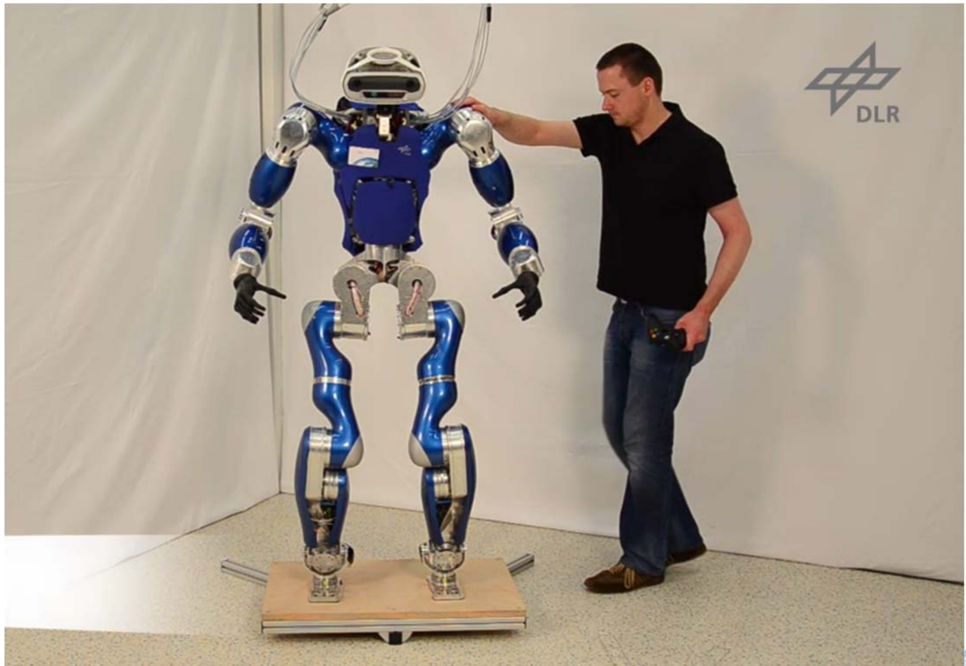
[Englsberger et al, Humanoids 2014]

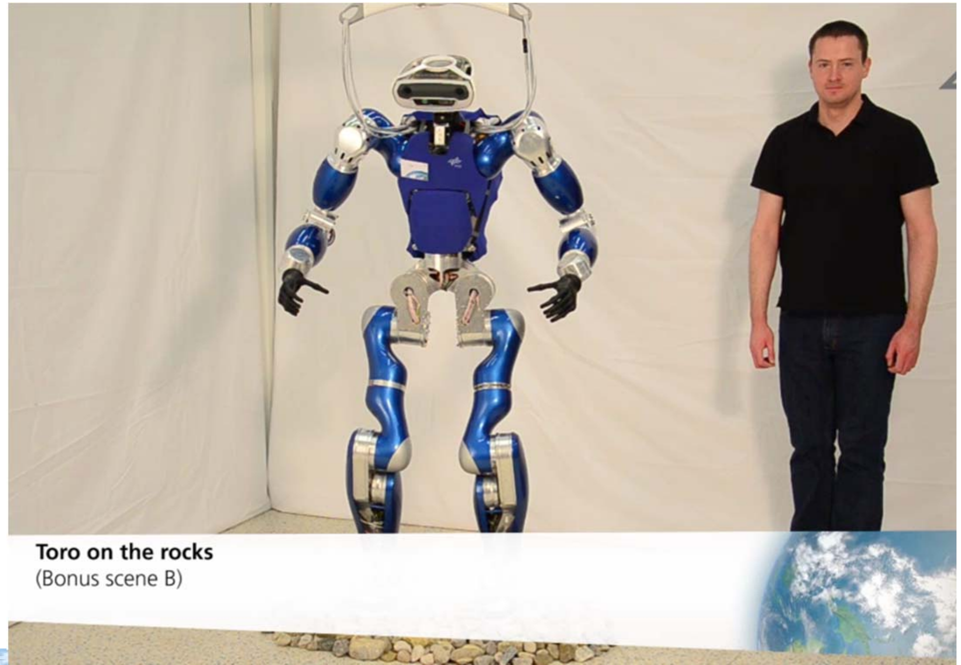
DLR


DLR
- Height: 1.74 m
- Mass: 76.4 kg
- Battery duration: approx. 1 hour
- 25 Joints can be operated in position and torque controlled mode (legs, arms, waist). Joints are based on the DLR-KUKA-Lightweight-Arm III
- 2 Joints are operated in position controlled mode (neck)
- Prostetic hands with 12 DoF in total


DLR




DLR


Toro on the rocks
(Bonus scene B)

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1) Humanoid robot TORO

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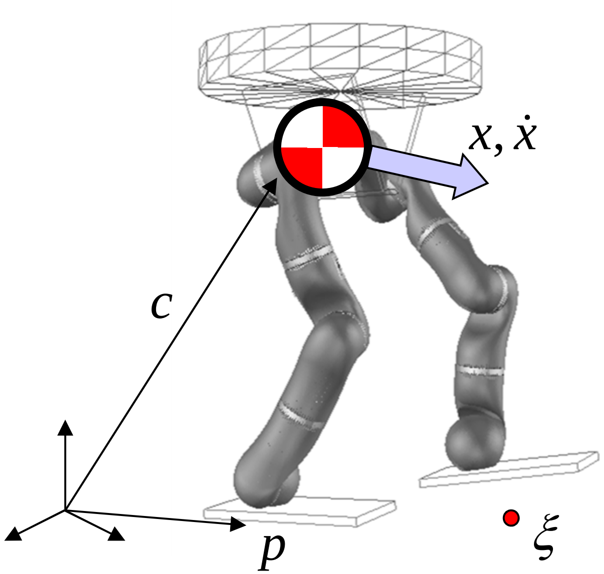
Walking Stabilization

[Englsberger, Ott, IROS 2013]



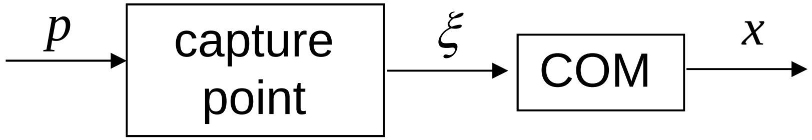
(Pratt 2006, Hof 2008)

Template model: $\ddot{x} = \omega^2 (x - p)$

$(x, \dot{x}) \downarrow (x, \xi)$

$$\xi = x + \frac{1}{\omega}\dot{x}$$

$$\dot{\xi} = \omega(\xi - p) \qquad \dot{x} = \omega(\xi - x)$$



open loop unstable

exp. stable

Walking Stabilization

[Englsberger, Ott, IROS 2013]

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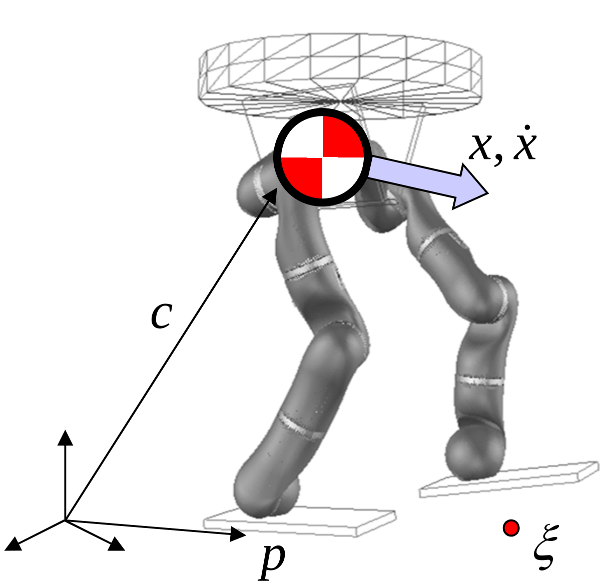

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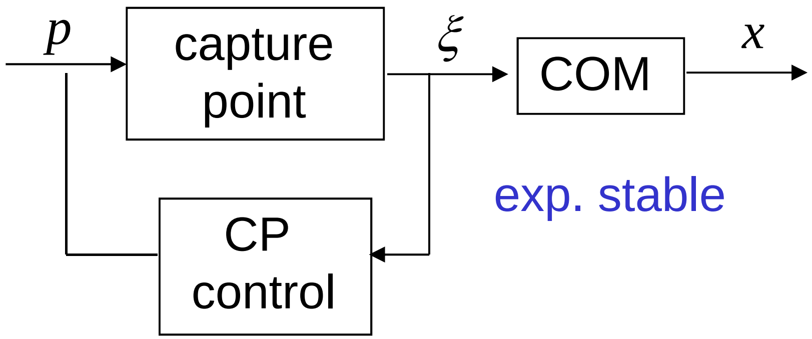
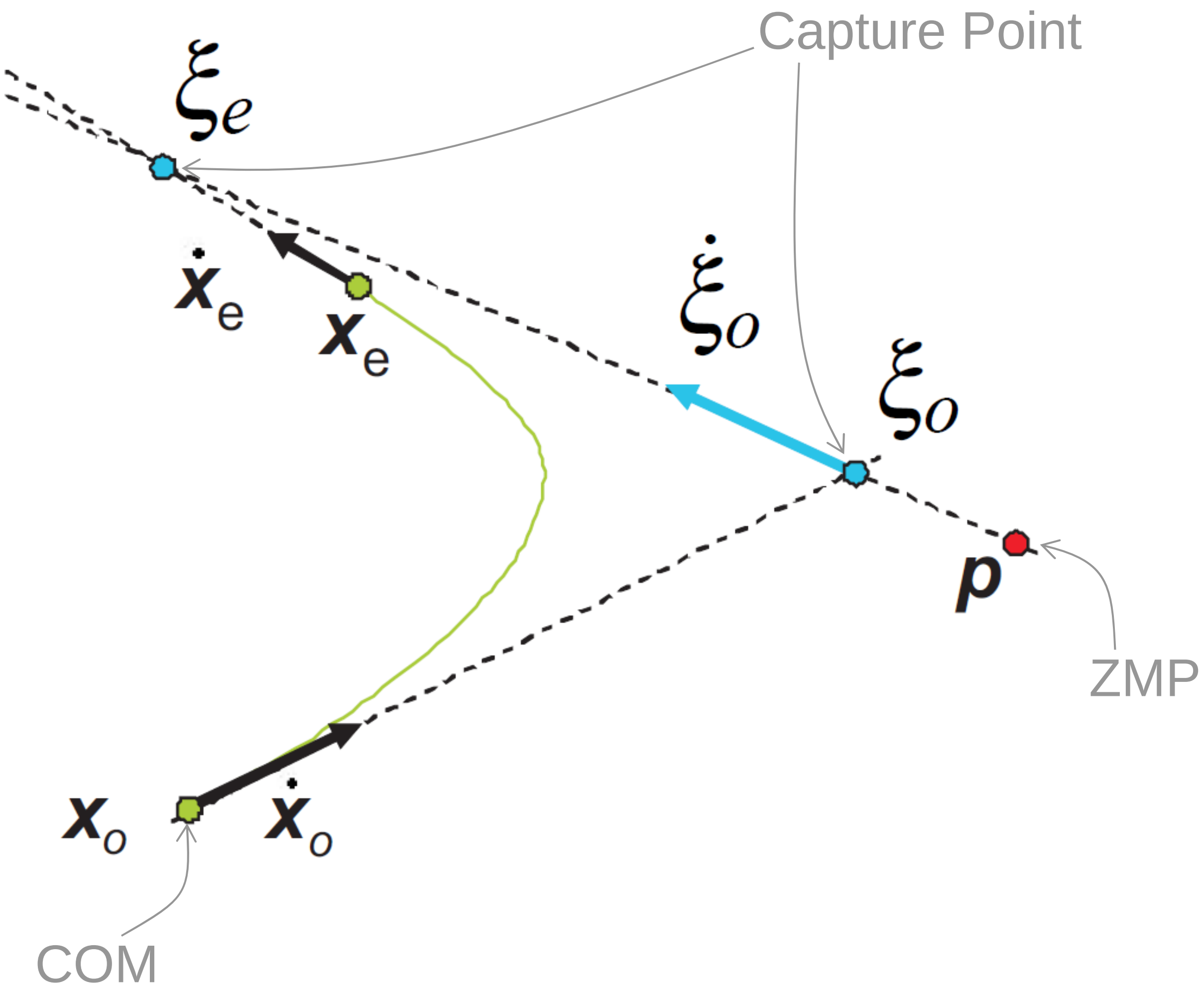


exp. stable

Using Capture Point for Walking

$$\dot{x} = -\omega x + \omega \xi \longleftrightarrow \xi = x + \frac{\dot{x}}{\omega}$$

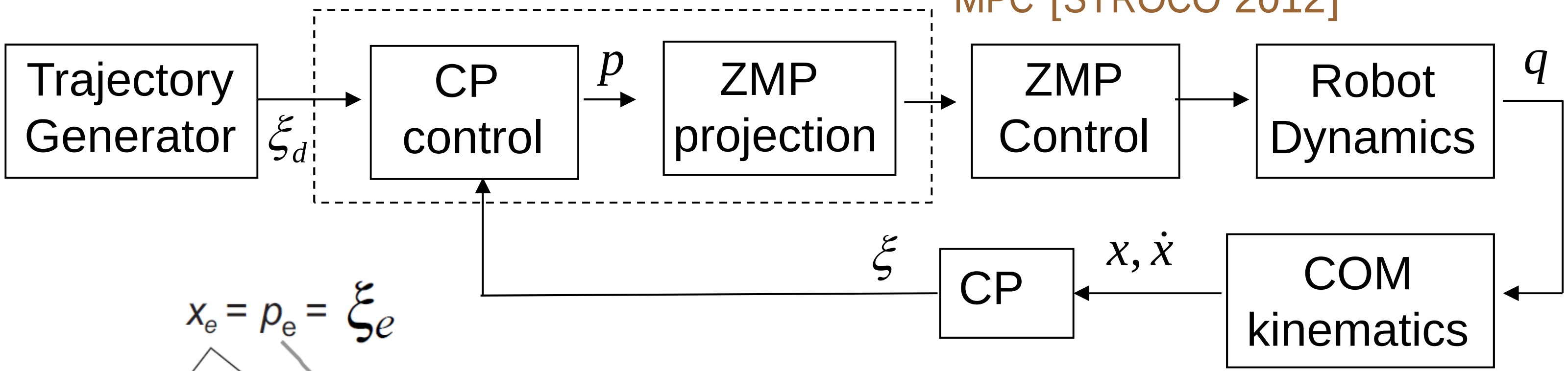
$$\dot{\xi} = \omega \xi - \omega p$$

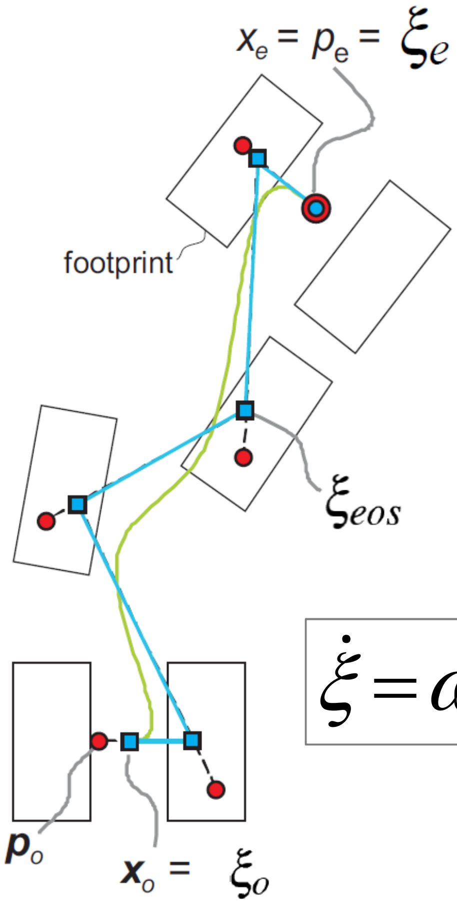


- COM velocity always points towards CP
- ZMP „pushes away“ the CP on a line
- COM follows CP

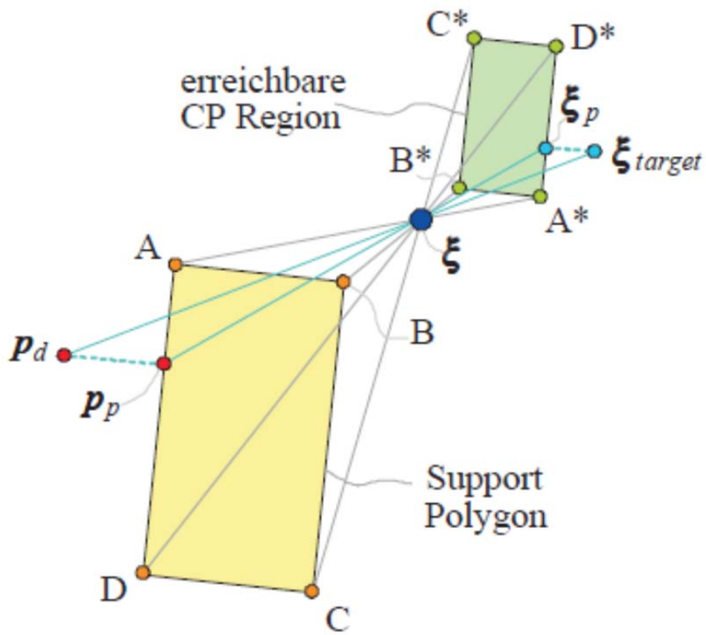
Capture Point Control

MPC [SYROCO 2012]



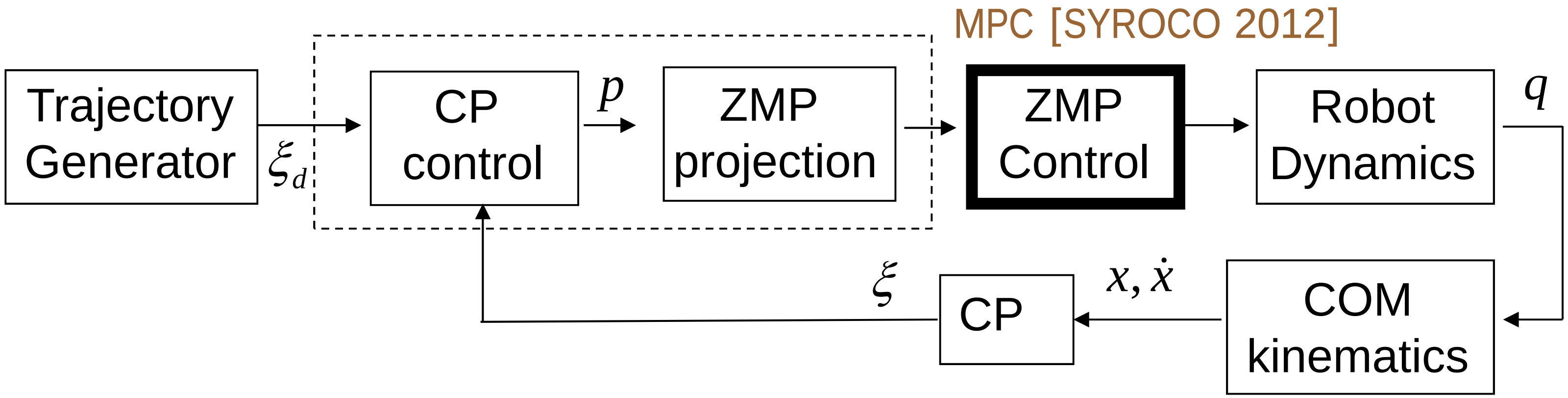


$$\dot{\xi} = \omega(\xi - p)$$



[Englsberger, Ott, et. al., IROS-2011, ICRA-2012, at-2012]

Position based ZMP Control



Desired ZMP implies a desired force acting on the COM:

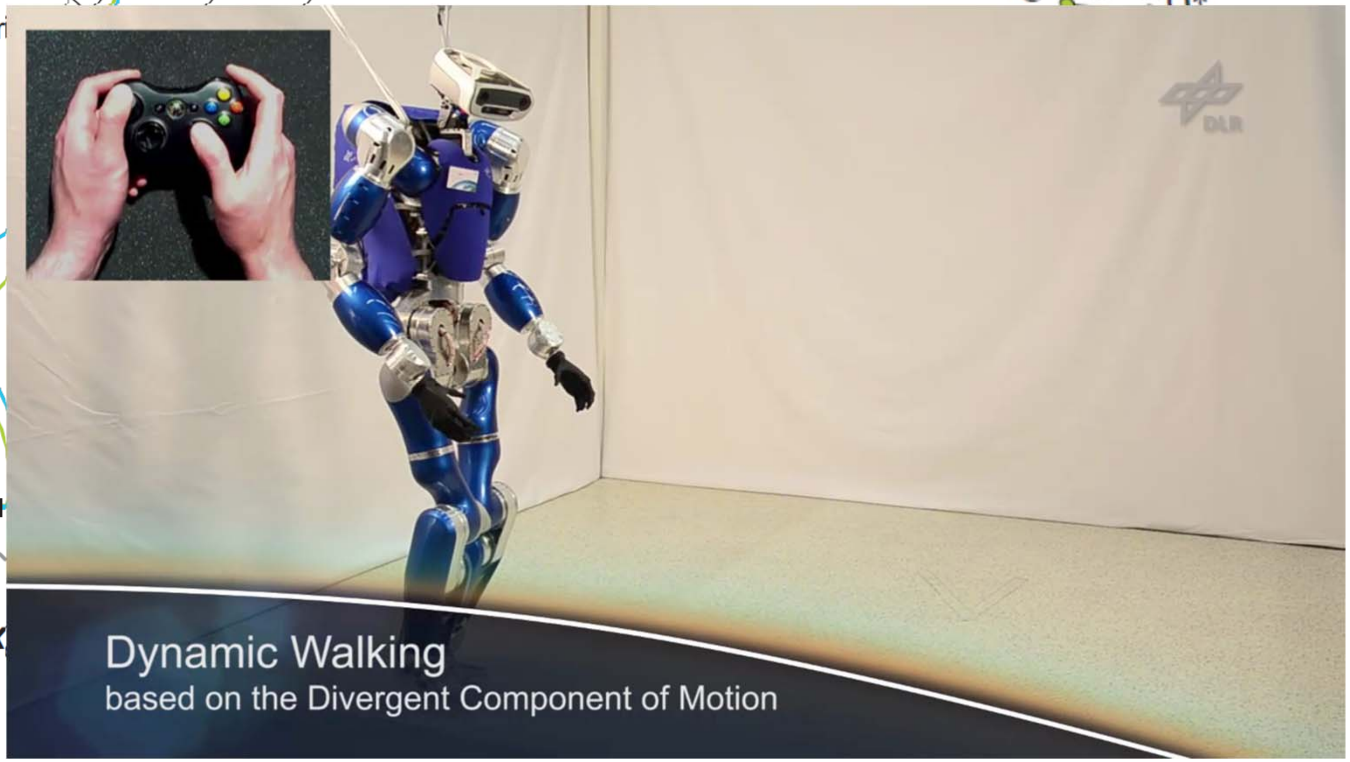
p_d $\quad \ddot{x} = \omega^2 (x - p) \quad \Rightarrow \quad$ $F_d = M\omega^2 (x - p_d)$

Position based ZMP Control

Position based force control [Roy&Whitcomb,2002]:

$\dot{x}_d = k_f M\omega^2 (p - p_d) \quad \Leftarrow \quad \dot{x}_d = k_f (F_d - F)$


Collaboration with Nicolas Perrin


Dynamic Walking
based on the Divergent Component of Motion

Extension to 3D walking

2D	3D
Capture Point (CP)	„Divergent Component of Motion“ (DCM) [Takenaka]
$\xi = x + b\dot{x}$	
ZMP (steers CP)	Virtual Repellent Point (steers DCM)

COM dynamics: $m\ddot{x} = F$
(not a template model)

$\uparrow$

$mg + F_{ext}$

DCM dynamics:

$$\dot{\xi} = -\frac{1}{b}x + \frac{1}{b}\xi + \frac{b}{m}\boldsymbol{F}$$

[Englsberger, Ott, IROS 2013]

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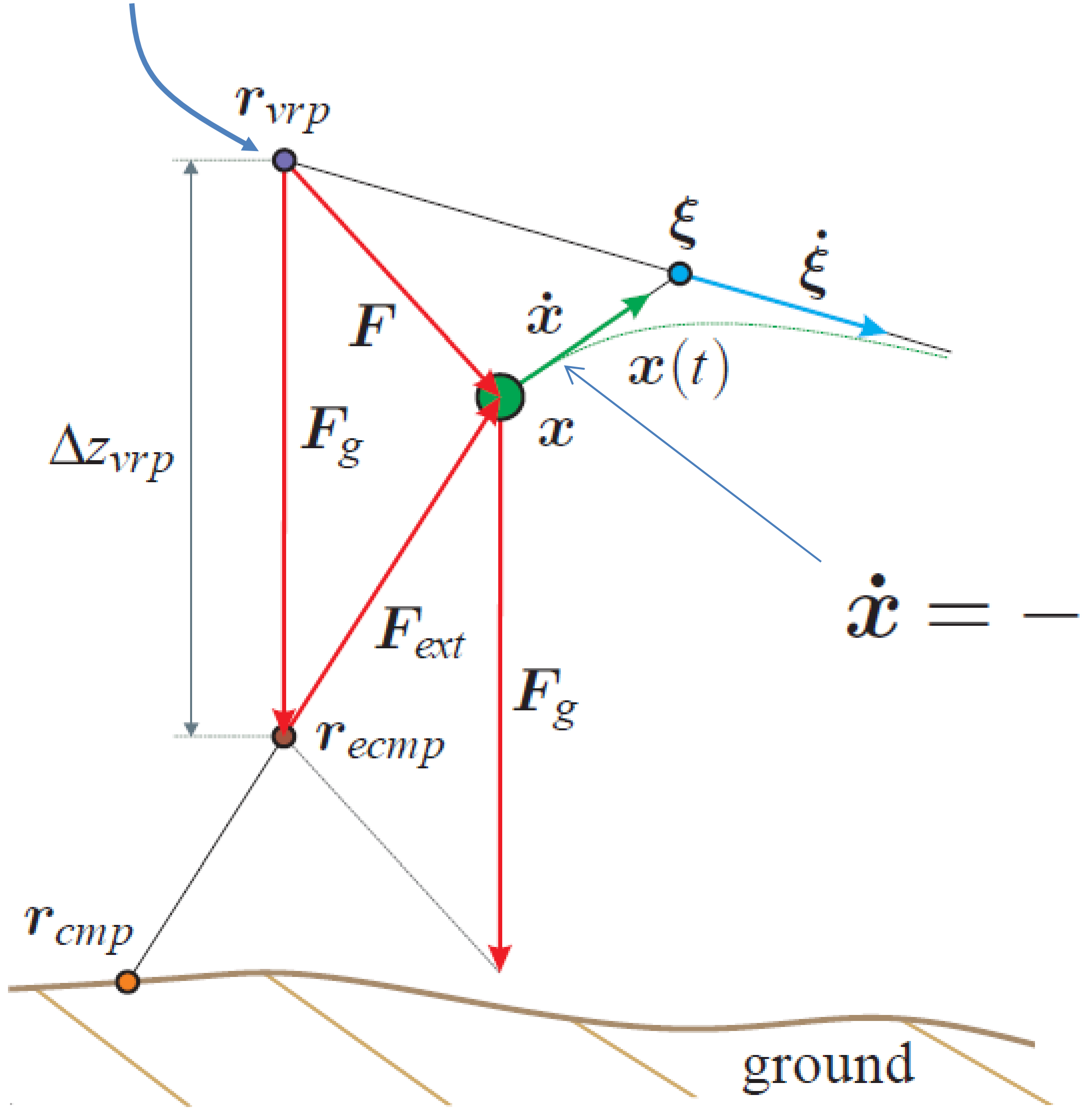
$$\dot{\xi} = -\frac{1}{b}x + \frac{1}{b}\xi + \frac{b}{m}F$$

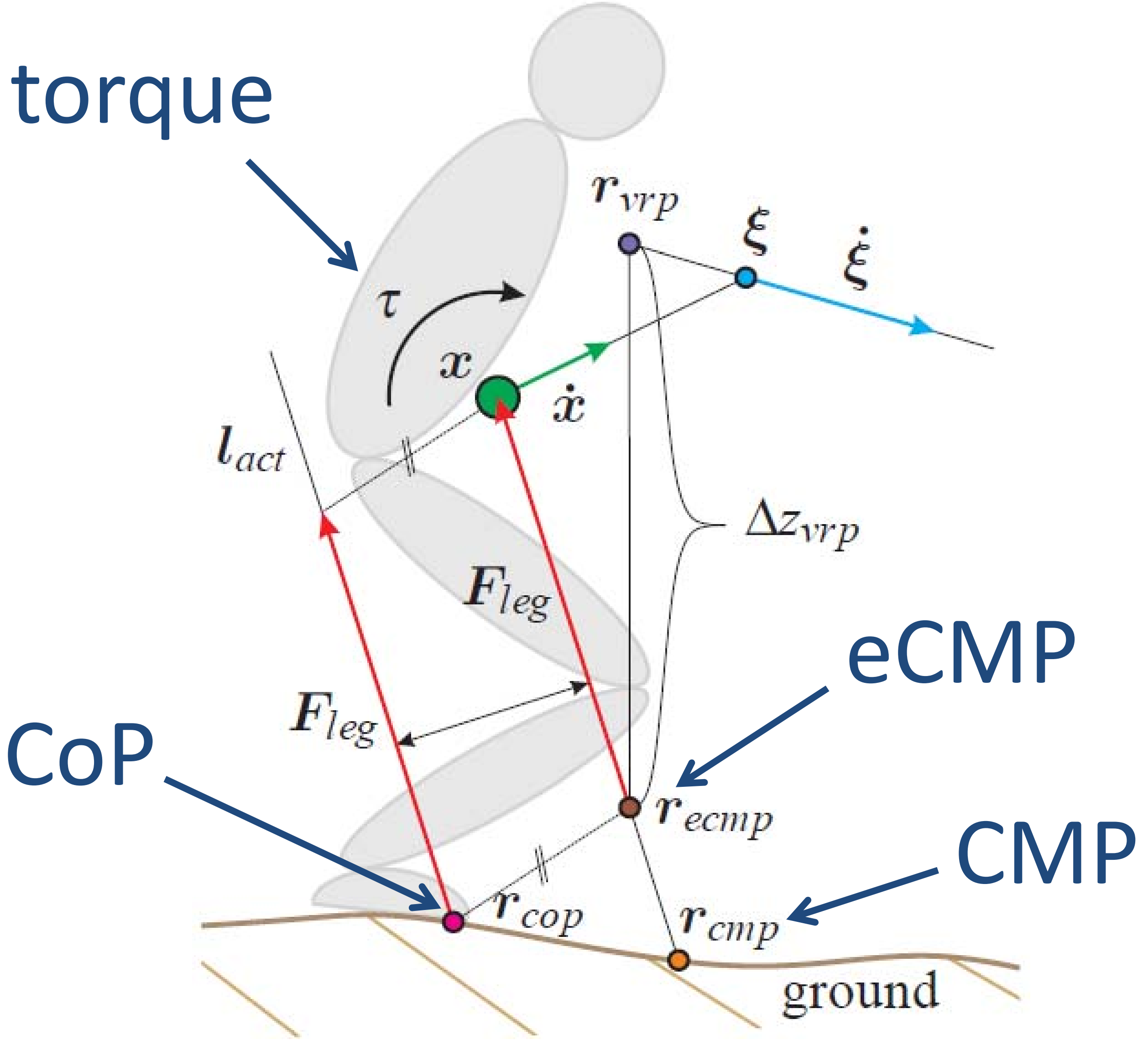
r_{vrp}

[Englsberger, Ott, IROS 2013]

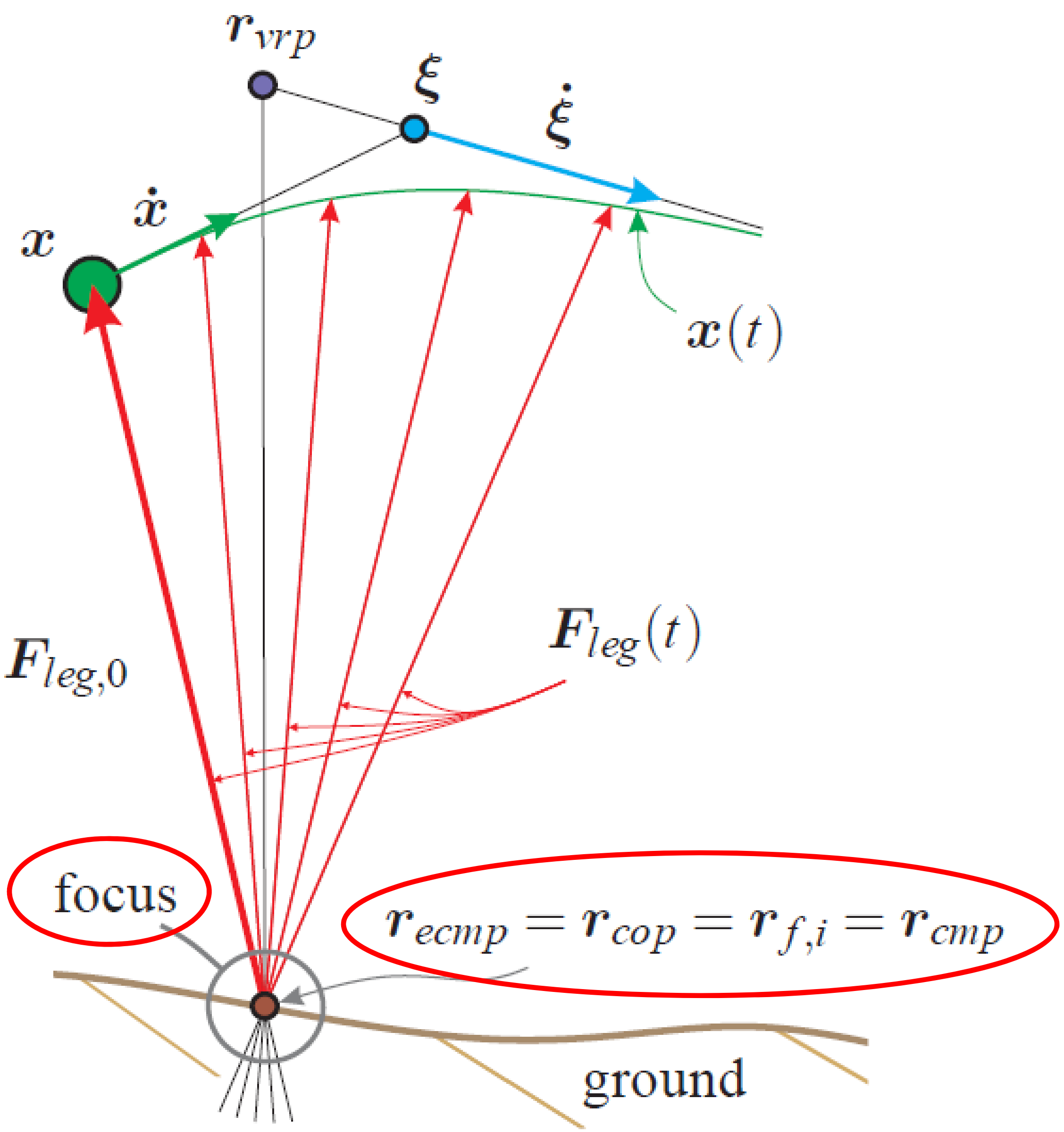
Virtual Repellent Point (VRP)

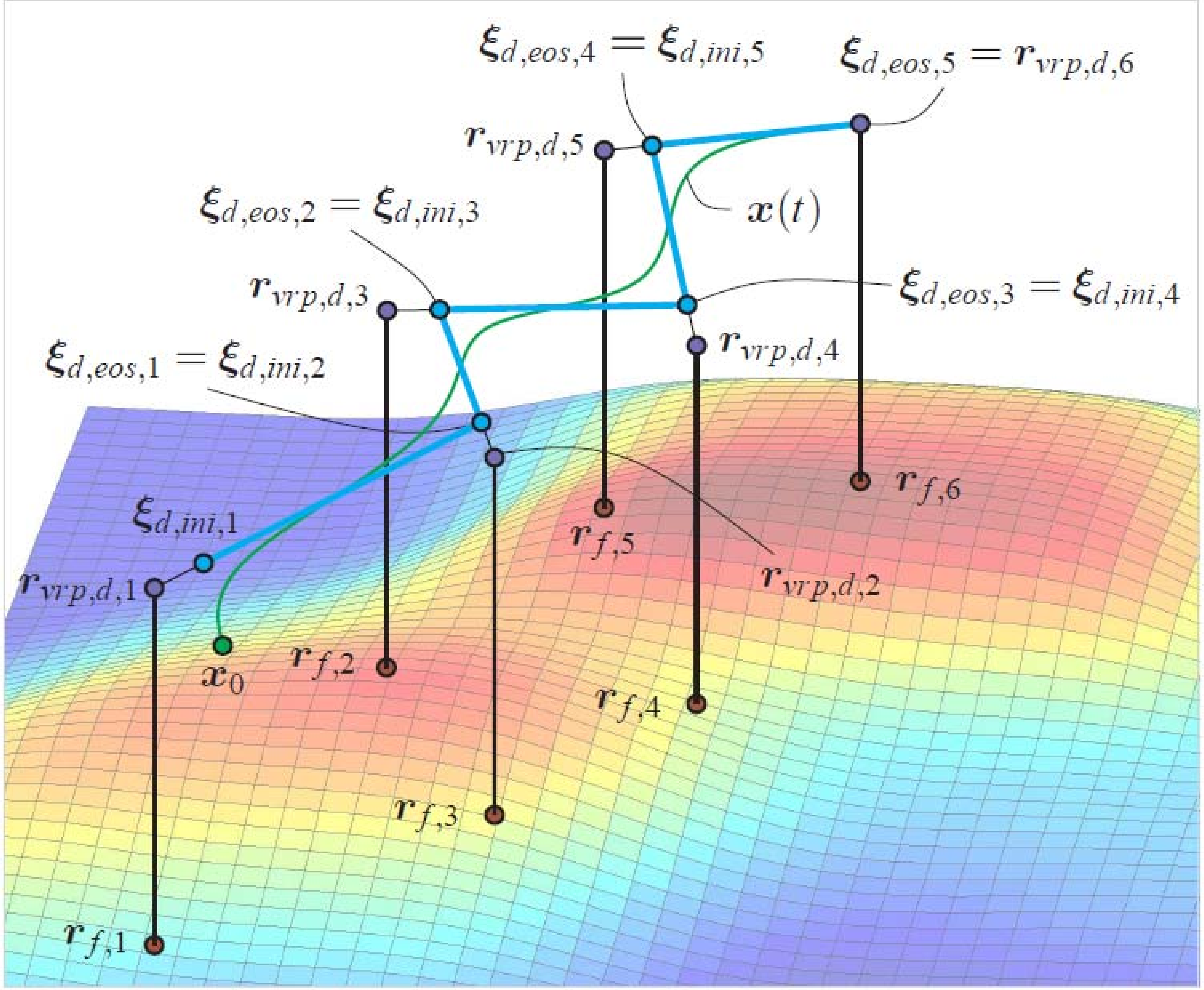


$$\dot{x} = -\frac{1}{b}(x - \xi)$$


torque
τ
x
ẋ
l_act
r_vrp
ξ
ξ̇
Δz_vrp
F_leg
F_leg
eCMP
r_ecmp
CoP
r_cop
CMP
r_cmp
ground

DCM trajectory generation




$\xi_{d,eos,4} = \xi_{d,ini,5}$
$\xi_{d,eos,5} = r_{vrp,d,6}$
$r_{vrp,d,5}$
$x(t)$
$\xi_{d,eos,2} = \xi_{d,ini,3}$
$r_{vrp,d,3}$
$\xi_{d,eos,3} = \xi_{d,ini,4}$
$r_{vrp,d,4}$
$\xi_{d,eos,1} = \xi_{d,ini,2}$
$r_{f,6}$
$\xi_{d,ini,1}$
$r_{f,5}$
$r_{vrp,d,1}$
$r_{vrp,d,2}$
$r_{f,2}$
x_0
$r_{f,4}$
$r_{f,3}$
$r_{f,1}$

DCM Tracking Control

DCM dynamics

$$\dot{\boldsymbol{\xi}} = \frac{1}{b}\left(\boldsymbol{\xi} - \boldsymbol{r}_{vrp}\right)$$

Desired closed loop

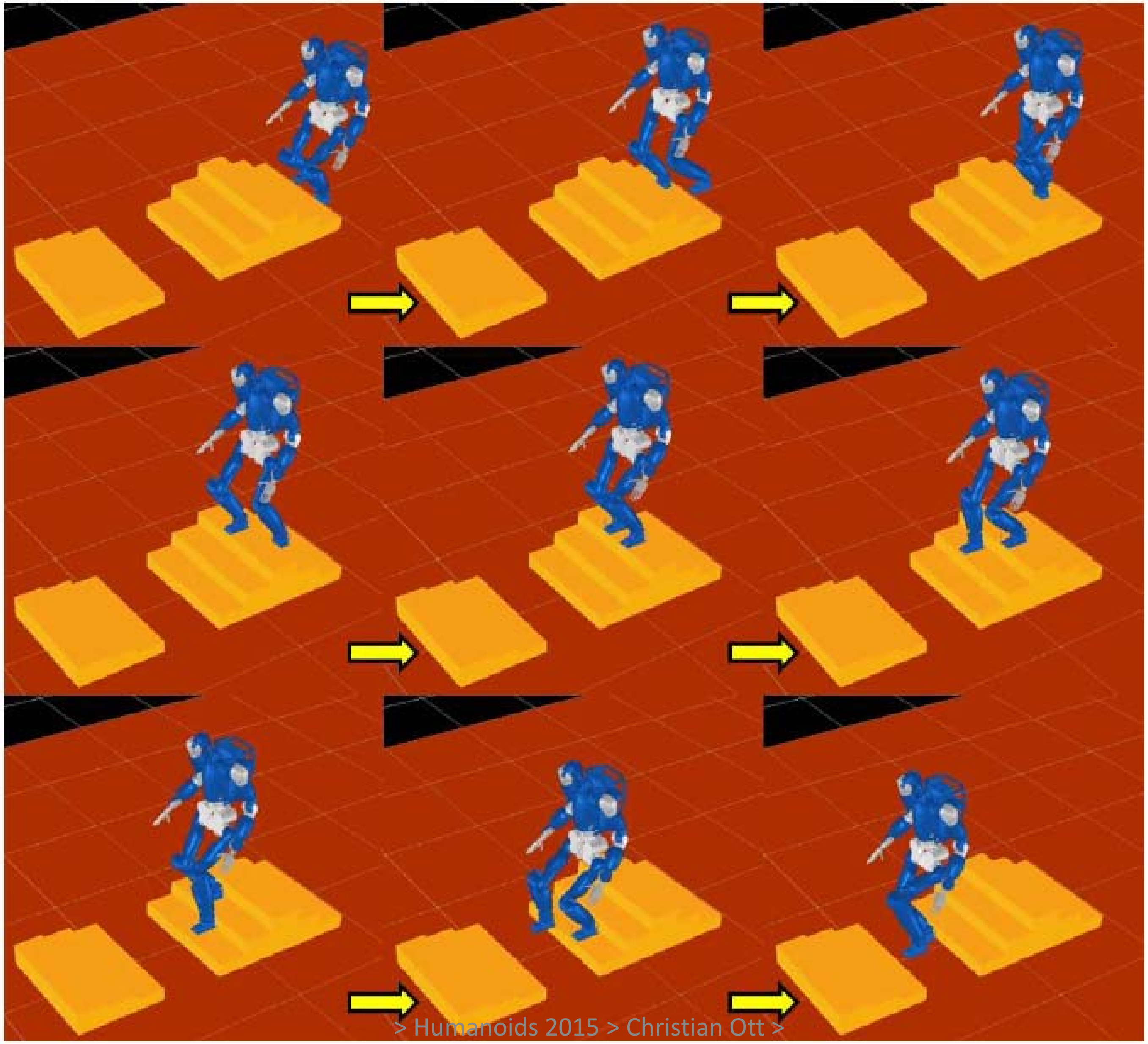
$$\underbrace{\dot{\boldsymbol{\xi}} - \dot{\boldsymbol{\xi}}_d}_{\dot{\boldsymbol{e}}_\xi} = -k\,\underbrace{(\boldsymbol{\xi} - \boldsymbol{\xi}_d)}_{\boldsymbol{e}_\xi}$$

Tracking control: $\boldsymbol{r}_{vrp,c} = \boldsymbol{\xi} + k\,b(\boldsymbol{\xi} - \boldsymbol{\xi}_d) - b\dot{\boldsymbol{\xi}}_d$

Required leg force:

$$\boldsymbol{F}_{leg,c} = \frac{mg}{\Delta z_{vrp}}\left(\boldsymbol{x} - \underbrace{\left(\boldsymbol{r}_{vrp,c} - \left[0\ 0\ \Delta z_{vrp}\right]^T\right)}_{\boldsymbol{r}_{ecmp,c}}\right)$$

OpenHRP

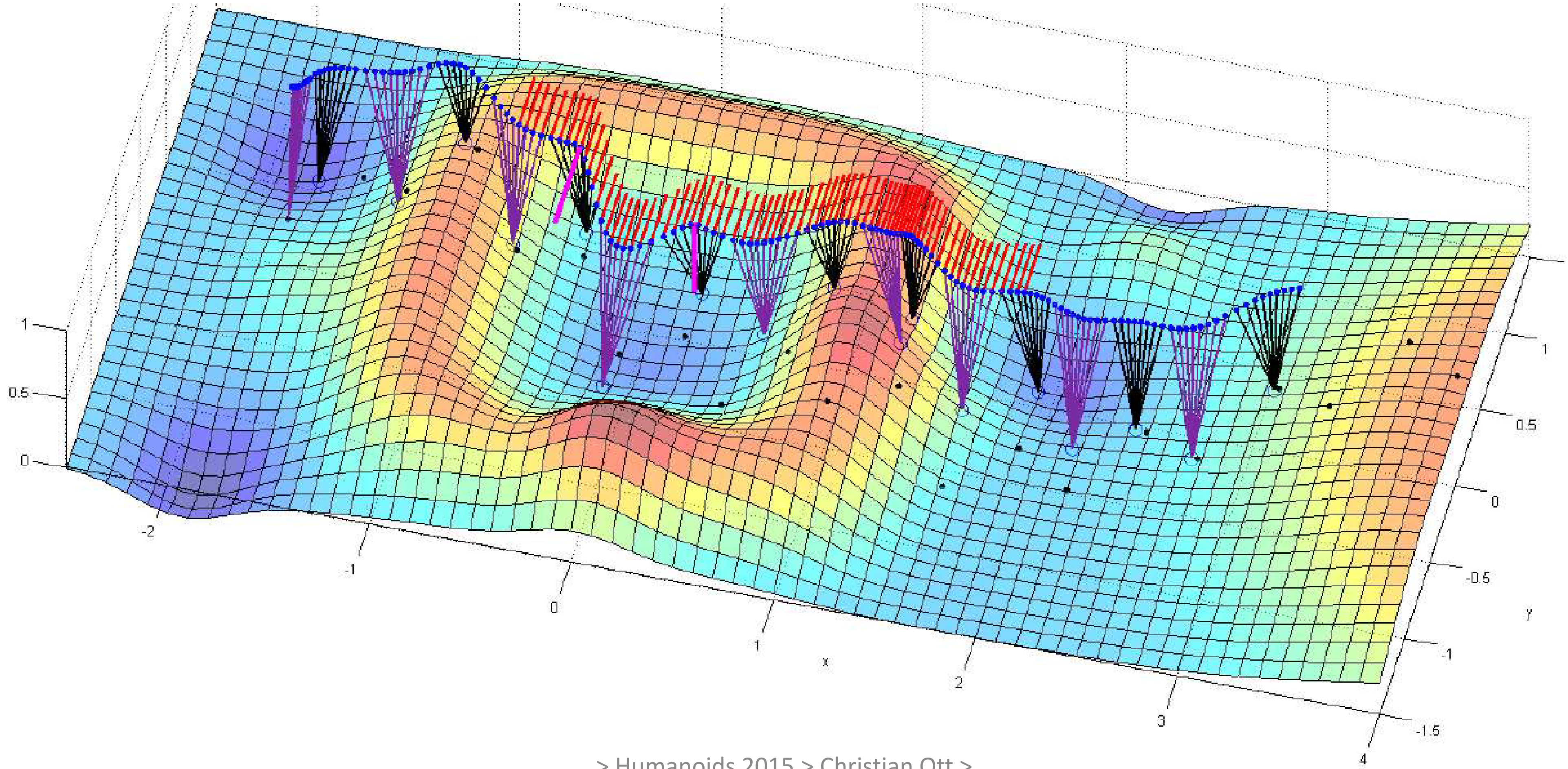


Simulation 2:
OpenHRP3 Simulation of TORO (DLR's bipedal humanoid)

Simulation parameters:

- step time: 1.25s
- max. stair height (varying stair height): 0.12 m
- frontal step length: 0.25 m

point mass simulation
(prismatic inverted pendulum model)



[Englsberger, Ott, IROS 2013]

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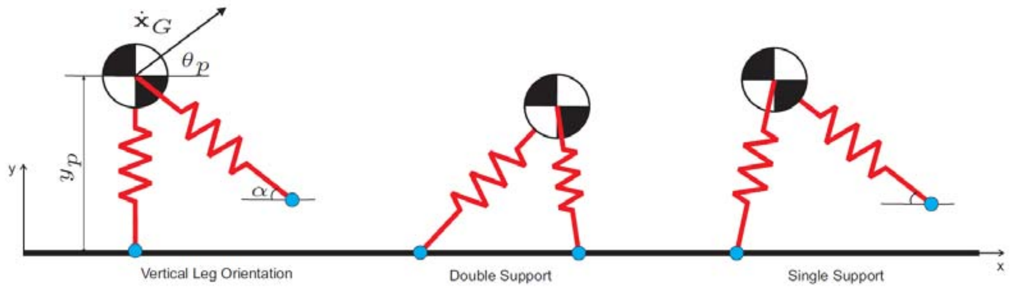
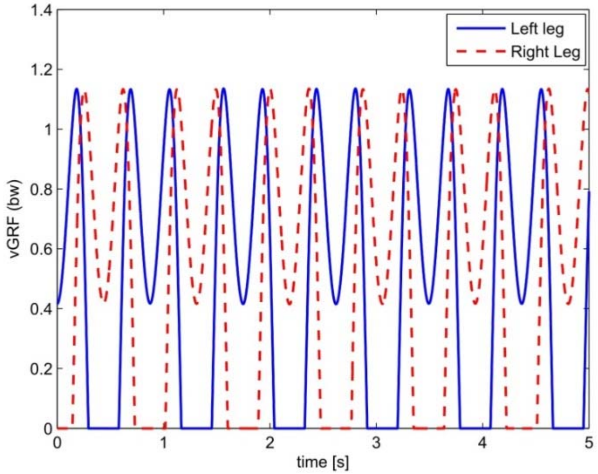
Humanoids 2015 Interactive Presentation by J. Englsberger

SLIP Template Model

Conceptual biomechanical model: single mass, mass-less legs, conservative



Vertical ground reaction force

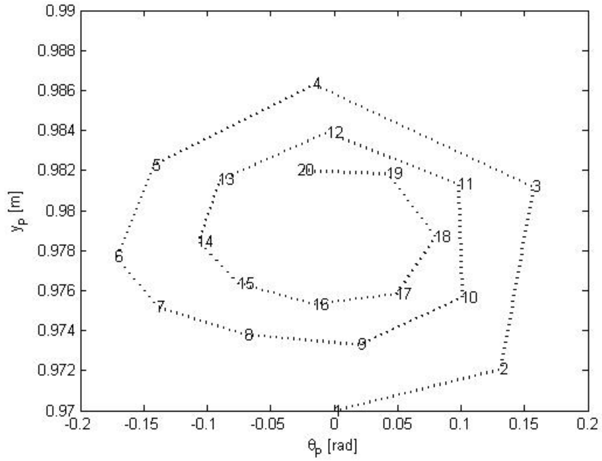


Mathematical model:

$$m\ddot{\mathbf{x}}_G = \mathbf{f}_R + \mathbf{f}_L + m\mathbf{g}_0$$

$$\mathbf{f}_i = k\left(\frac{l_0}{\|\mathbf{x}-\mathbf{x}_{F_i}\|} - 1\right)(\mathbf{x}-\mathbf{x}_{F_i})$$

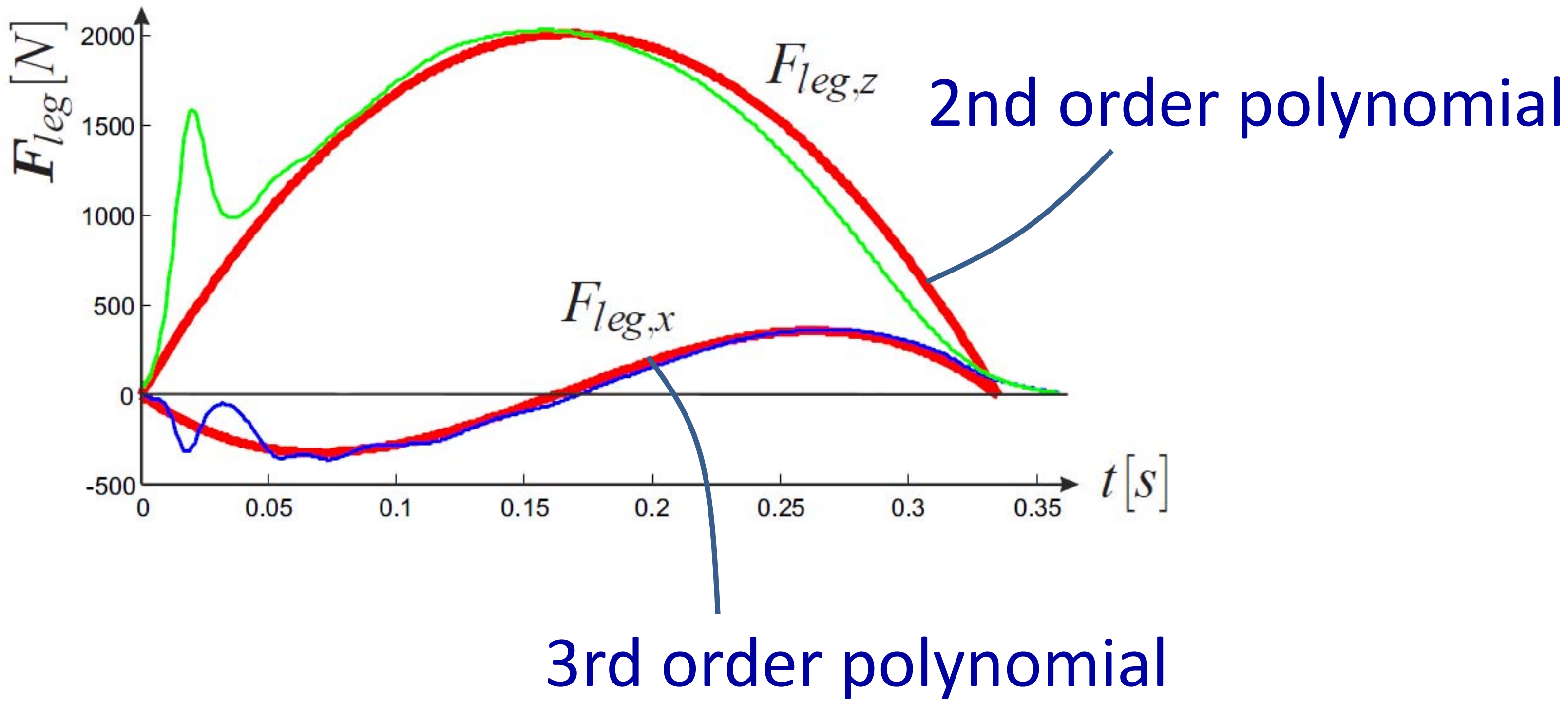
Poincare Map



✓ Existence of stable limit cycles can be shown

✓ Vertical ground reaction force resembles human data

Human experiments as motivation



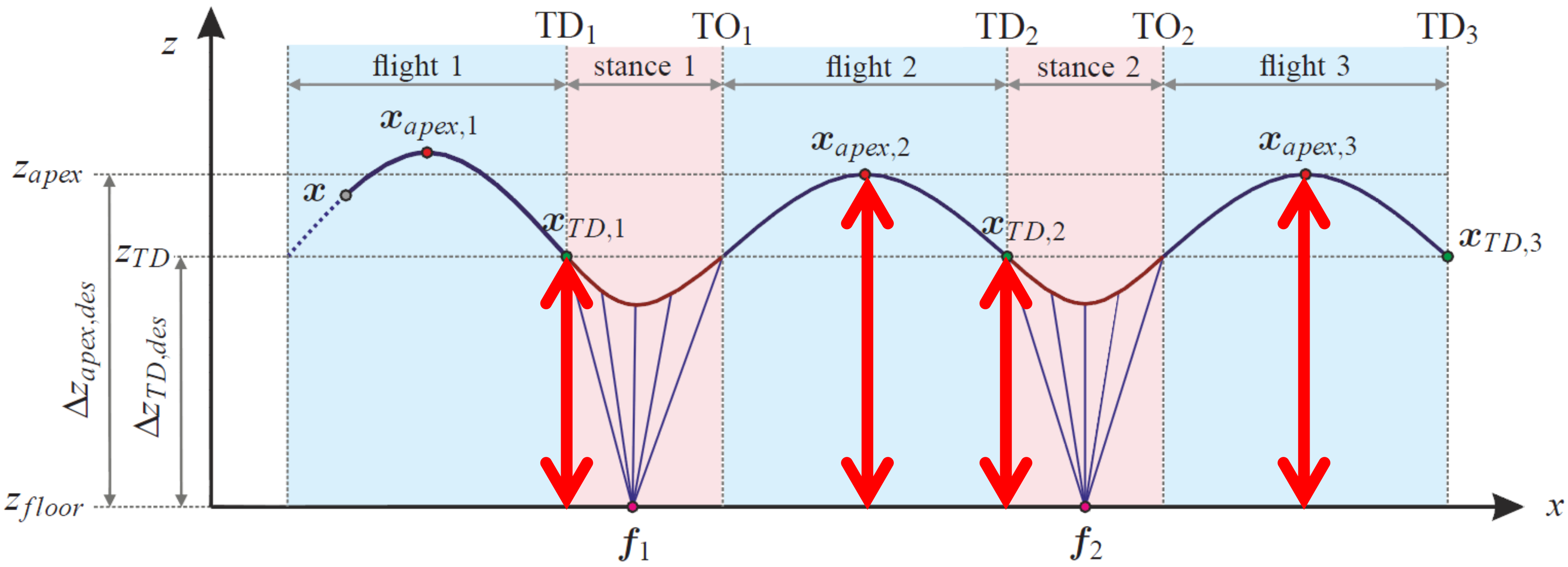
Force and motion encoding (during stance)

	vertical	**horizontal**
force	2nd order	3rd order
CoM position	4th order	5th order
	five parameters	**six parameters**

$$m\ddot{x} = F$$

$$\begin{bmatrix} \sigma(t) \\ \dot{\sigma}(t) \\ \ddot{\sigma}(t) \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & t & t^2 & t^3 & t^4 & t^5 \\ 0 & 1 & 2t & 3t^2 & 4t^3 & 5t^4 \\ 0 & 0 & 2 & 6t & 12t^2 & 20t^3 \end{bmatrix}}_{\begin{bmatrix} \mathbf{t}_{\sigma}^T(t) \\ \mathbf{t}_{\dot{\sigma}}^T(t) \\ \mathbf{t}_{\ddot{\sigma}}^T(t) \end{bmatrix}} \mathbf{p}_{\sigma}, \quad \sigma \in \{x, y, z\}$$

Preview / Planning



design parameters

- touch-down height
- apex height
- time of stance

Flight Dynamics

$$\boldsymbol{x}(t) = \boldsymbol{x}_0 + \dot{\boldsymbol{x}}_0\, t + \boldsymbol{g}\, \frac{t^2}{2}$$

$$\dot{\boldsymbol{x}}(t) = \dot{\boldsymbol{x}}_0 + \boldsymbol{g}\, t$$

$$\Delta t_{apex} = \frac{\dot{z}}{g}$$

$$\Delta t_{TD} = \Delta t_{apex} + \sqrt{\Delta t_{apex}^2 + \frac{2}{g}(z - z_{TD})}$$

Vertical planning (five parameters)

$$\underbrace{\begin{bmatrix} z_{TD} \\ \dot{z}_{TD} \\ -g \\ -g \end{bmatrix}}_{\boldsymbol{b}_z} = \underbrace{\begin{bmatrix} \boldsymbol{t}_z^T(0) \\ \boldsymbol{t}_{\dot{z}}^T(0) \\ \boldsymbol{t}_{\ddot{z}}^T(0) \\ \boldsymbol{t}_{\ddot{z}}^T(T_s) \end{bmatrix}}_{\boldsymbol{B}_z} \boldsymbol{p}_z$$

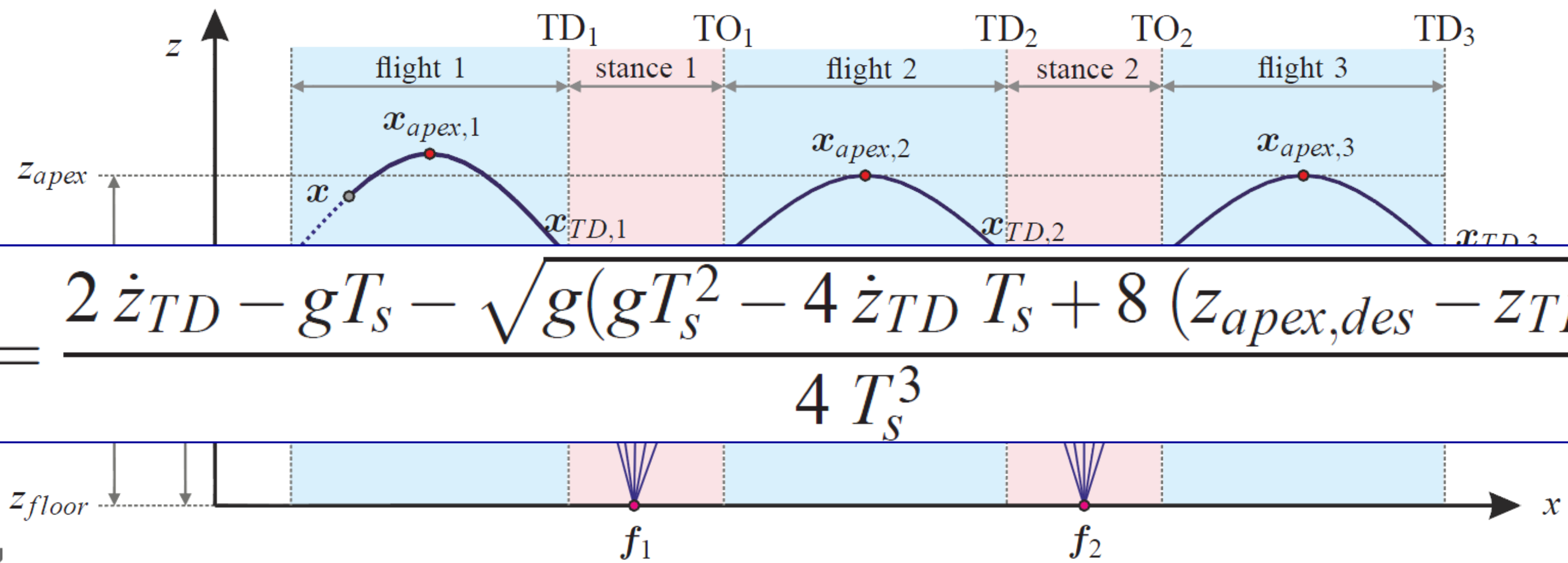
$$\boldsymbol{p}_z = \boldsymbol{B}_z^T (\boldsymbol{B}_z \boldsymbol{B}_z^T)^{-1} \boldsymbol{b}_z + \boldsymbol{r}_z \tilde{p}_z$$

Vertical planning
=> achieving apex height

$$z_{TO} = \boldsymbol{t}_z^T(T_s)\,\boldsymbol{p}_z$$

$$\dot{z}_{TO} = \boldsymbol{t}_{\dot{z}}^T(T_s)\,\boldsymbol{p}_z$$

$$z_{apex} = z_{TO} + \frac{\dot{z}_{TO}^2}{2g}$$



$$\tilde{p}_z = \frac{2\,\dot{z}_{TD} - gT_s - \sqrt{g(gT_s^2 - 4\,\dot{z}_{TD}\,T_s + 8\,(z_{apex,des} - z_{TD}))}}{4\,T_s^3}$$

Horizontal planning (six parameters)

$$\begin{bmatrix} \chi_{TD} \\ \dot{\chi}_{TD} \\ 0 \\ 0 \\ \chi \dots \end{bmatrix} = \underbrace{\begin{bmatrix} \boldsymbol{t}_{\chi}^{T}(0) \\ \boldsymbol{t}_{\dot{\chi}}^{T}(0) \\ \boldsymbol{t}_{\ddot{\chi}}^{T}(0) \\ \boldsymbol{t}_{\ddot{\chi}}^{T}(T_s) \\ \boldsymbol{t}_{\chi}^{T}(T_s) + T_f\, \boldsymbol{t}_{\dot{\chi}}^{T}(T_s) \end{bmatrix}}_{\boldsymbol{B}_{\chi}} \boldsymbol{p}_{\chi}$$

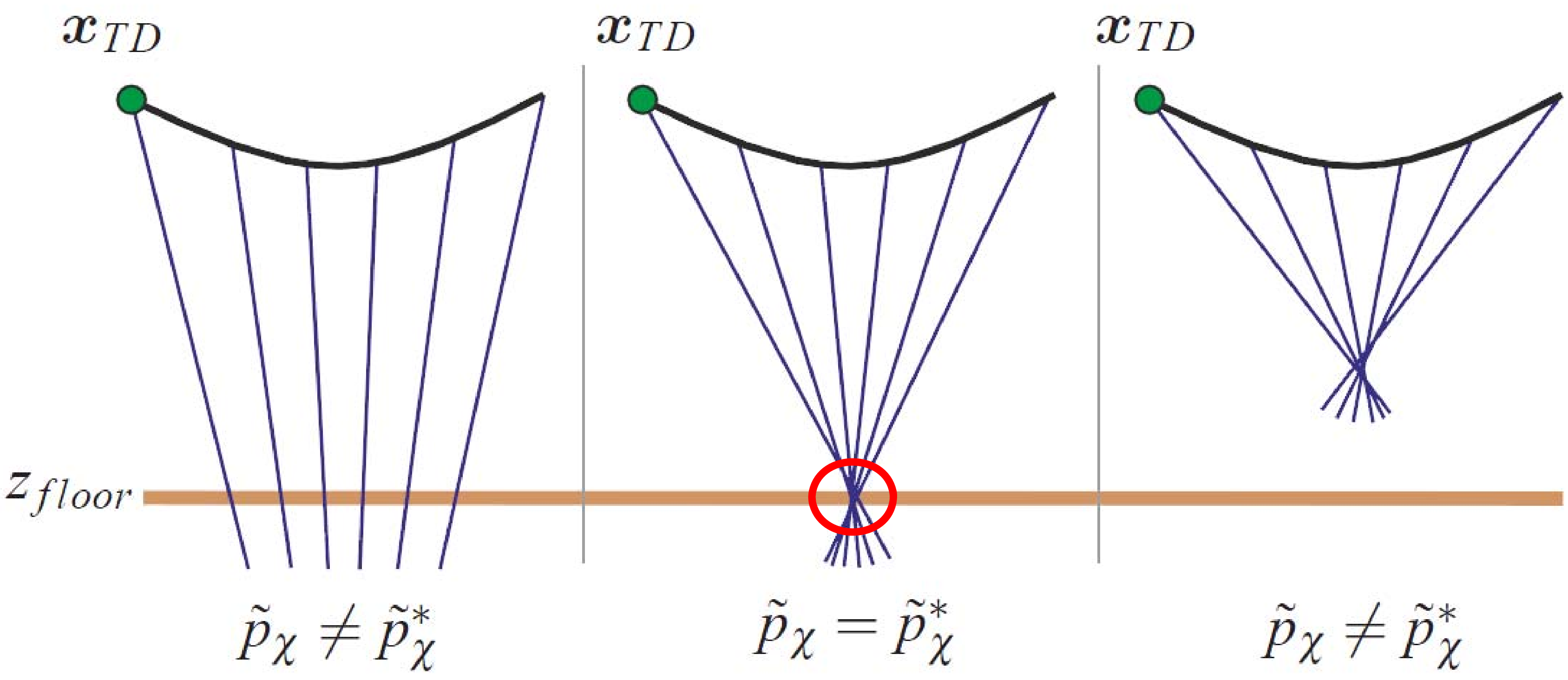
??

$$[\dots]\boldsymbol{B}_{\chi}^{T})^{-1}\, \boldsymbol{b}_{\chi} + \boldsymbol{r}_{\chi}\, \tilde{p}_{\chi}$$

+ force ray focusing

(quadratic)

Force ray focusing



least deviation/variance

(=> CoP …)

Minimizing variance ….

$$
\begin{aligned}
\chi_{int}(t_s) &= \chi(t_s) - \frac{f_{leg,\chi}(t_s)}{f_{leg,z}(t_s)}\left(z(t_s) - z_{floor}\right) \\
&= \underbrace{\left(\boldsymbol{t}_{\chi}^{T}(t_s) - \frac{\left(\boldsymbol{t}_{z}^{T}(t_s)\boldsymbol{p}_z - z_{floor}\right)\boldsymbol{t}_{\ddot{\chi}}^{T}(t_s)}{\boldsymbol{t}_{\ddot{z}}^{T}(t_s)\boldsymbol{p}_z + g}\right)}_{\boldsymbol{d}^{T}(t_s)} \boldsymbol{p}_{\chi}
\end{aligned}
$$

$$
\overline{\chi}_{int} = \frac{1}{T_s}\int_{t_s=0}^{T_s} \chi_{int}(t_s)\, dt_s = \underbrace{\frac{1}{T_s}\int_{t_s=0}^{T_s} \boldsymbol{d}^{T}(t_s)\, dt_s}_{\boldsymbol{e}^{T}} \boldsymbol{p}_{\chi}
$$

Minimizing variance ….

$$\Delta\chi_{int}(t_s) = \chi_{int}(t_s) - \overline{\chi}_{int} = \underbrace{\left(\mathbf{d}^T(t_s) - \mathbf{e}^T\right)}_{\boldsymbol{k}^T(t_s)} \boldsymbol{p}_\chi$$

$$\Delta\chi^2_{int}(t_s) = \boldsymbol{p}^T_\chi \boldsymbol{k}(t_s)\, \boldsymbol{k}^T(t_s) \boldsymbol{p}_\chi = \boldsymbol{p}^T_\chi \boldsymbol{L}(t_s)\, \boldsymbol{p}_\chi$$

Minimizing variance
(mean square deviation)….

• 36

$$\chi_{int,ms} = \boldsymbol{p}_\chi^T \frac{1}{T_s}\int_{t_s=0}^{T_s} \boldsymbol{L}(t_s)\, dt_s\, \boldsymbol{p}_\chi = \boldsymbol{p}_\chi^T\, \boldsymbol{M}\, \boldsymbol{p}_\chi$$

$$= \underbrace{\boldsymbol{r}_\chi^T \boldsymbol{M}\, \boldsymbol{r}_\chi}_{\alpha}\, \tilde{p}_\chi^2 + \underbrace{2\, \boldsymbol{r}_\chi^T \boldsymbol{M} \boldsymbol{p}_{\chi,0}}_{\beta}\, \tilde{p}_\chi + \underbrace{\boldsymbol{p}_{\chi,0}^T \boldsymbol{M} \boldsymbol{p}_{\chi,0}}_{\gamma}$$

scalar, but difficult to evaluate (non-linearities)

$$\chi_{int,ms} = \alpha\, \tilde{p}_\chi^2 + \beta\, \tilde{p}_\chi + \gamma$$

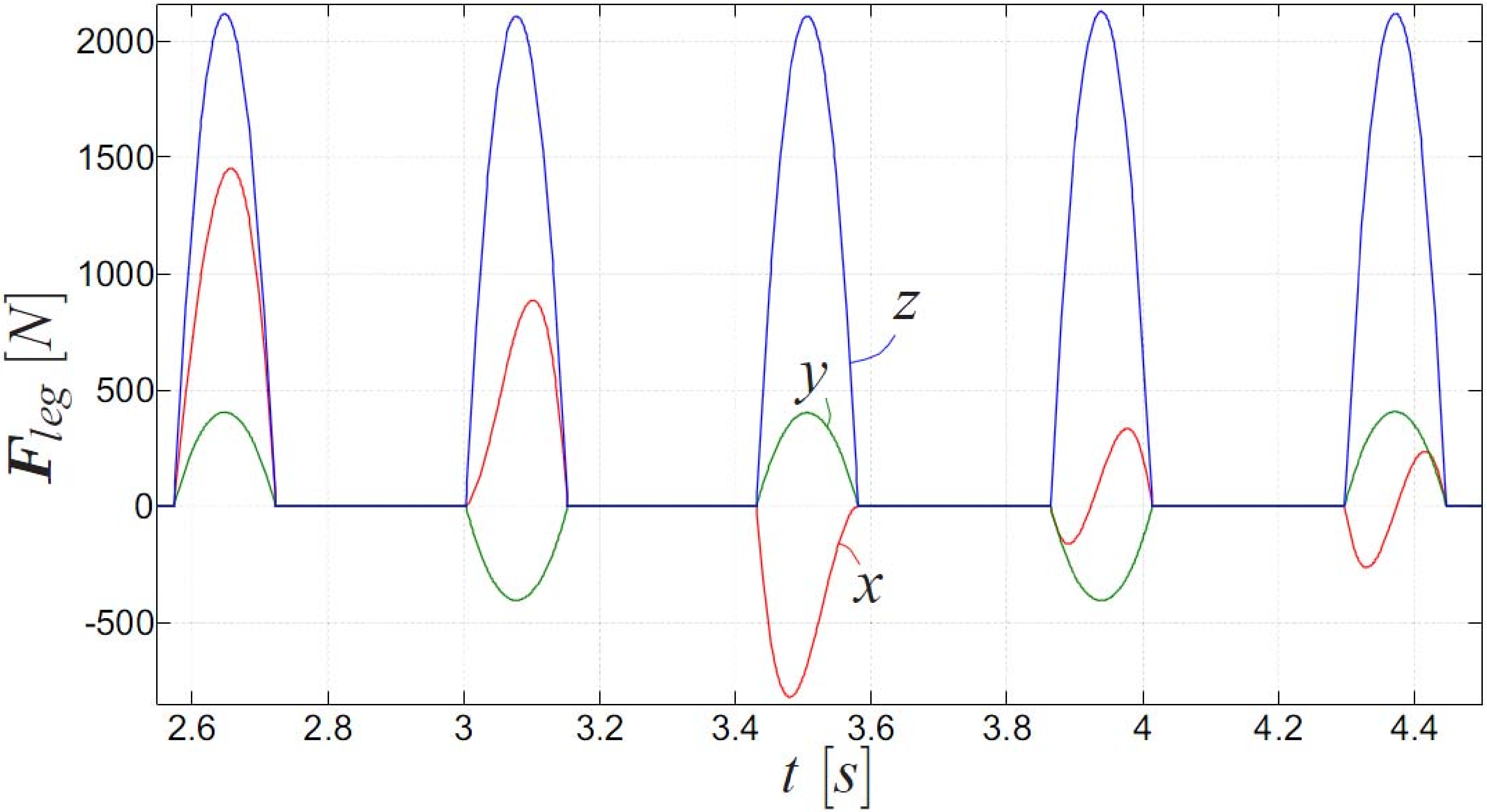
Leg Force evaluation

$$\boldsymbol{F}_{CoM,des}(t_s) = m \begin{bmatrix} \boldsymbol{t}_{\ddot{x}}^T(t_s)\,\boldsymbol{p}_x \\ \boldsymbol{t}_{\ddot{y}}^T(t_s)\,\boldsymbol{p}_y \\ \boldsymbol{t}_{\ddot{z}}^T(t_s)\,\boldsymbol{p}_z \end{bmatrix}$$

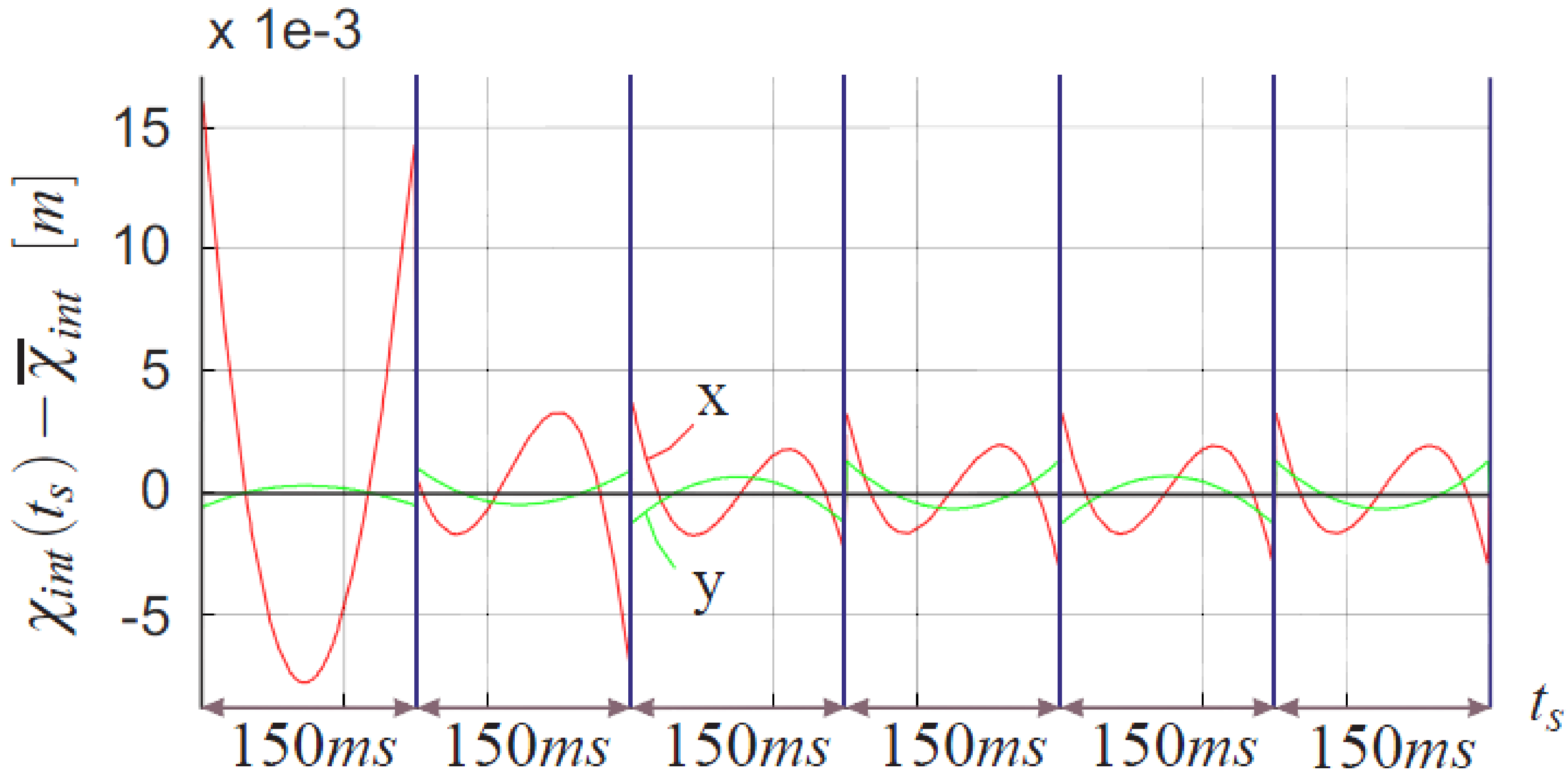
$$\boldsymbol{F}_{leg,des} = \boldsymbol{F}_{CoM,des} - \boldsymbol{F}_g$$

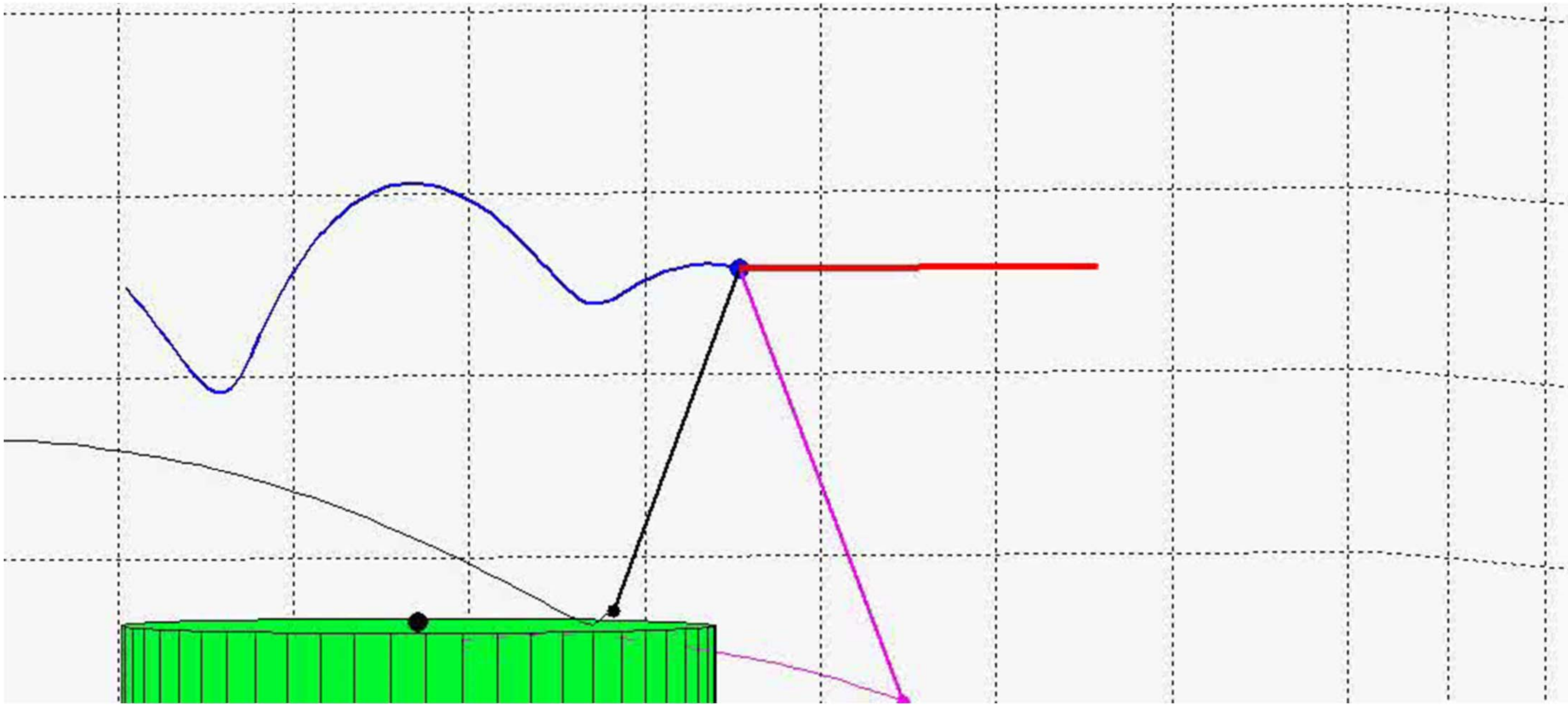
Typical force profiles

•



Deviation from point-foot (if not projected)





Summary

1) Walking Control based on the Capture Point
2) Extension to 3D
3) Running via polynomial leg force design
4) Implementation requires leg force control