



**DLR**

**Deutsches Zentrum  
für Luft- und Raumfahrt**  
German Aerospace Center

# Balancing and walking control of a torque controlled humanoid robot

**Dr.-Ing. Christian Ott**

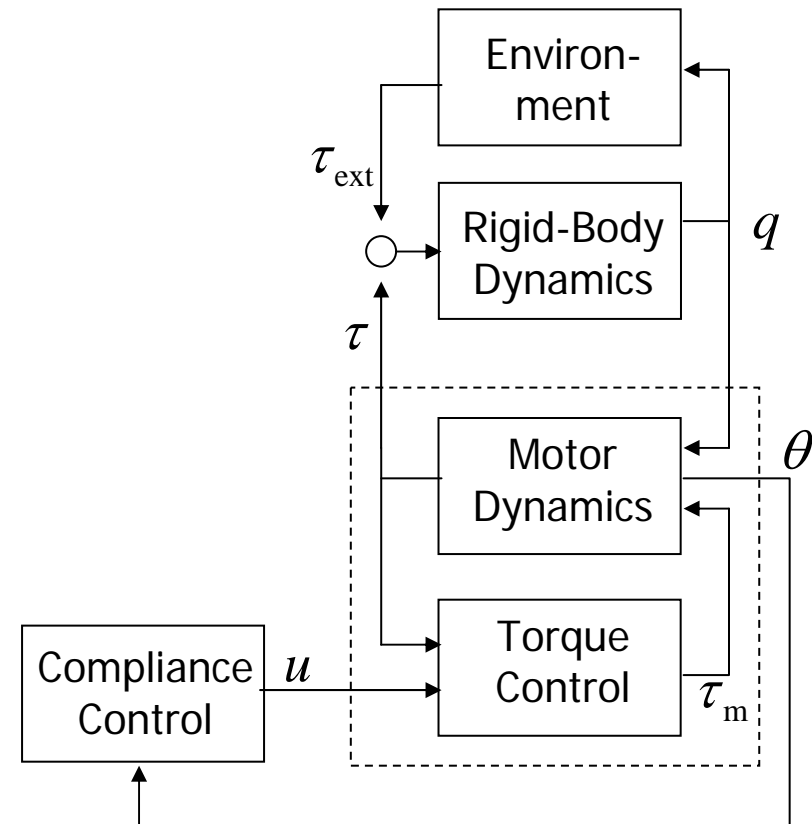
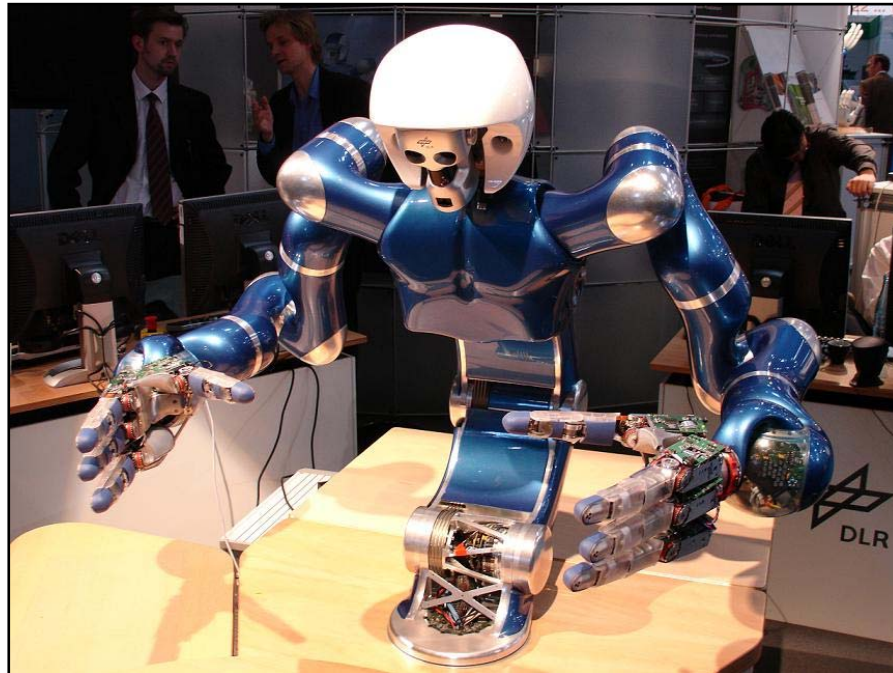
Research Group on „Dynamic Control of Legged Humanoid Robots“

DLR - Institute for Robotics and Mechatronics

Humanoids 29/11/2012



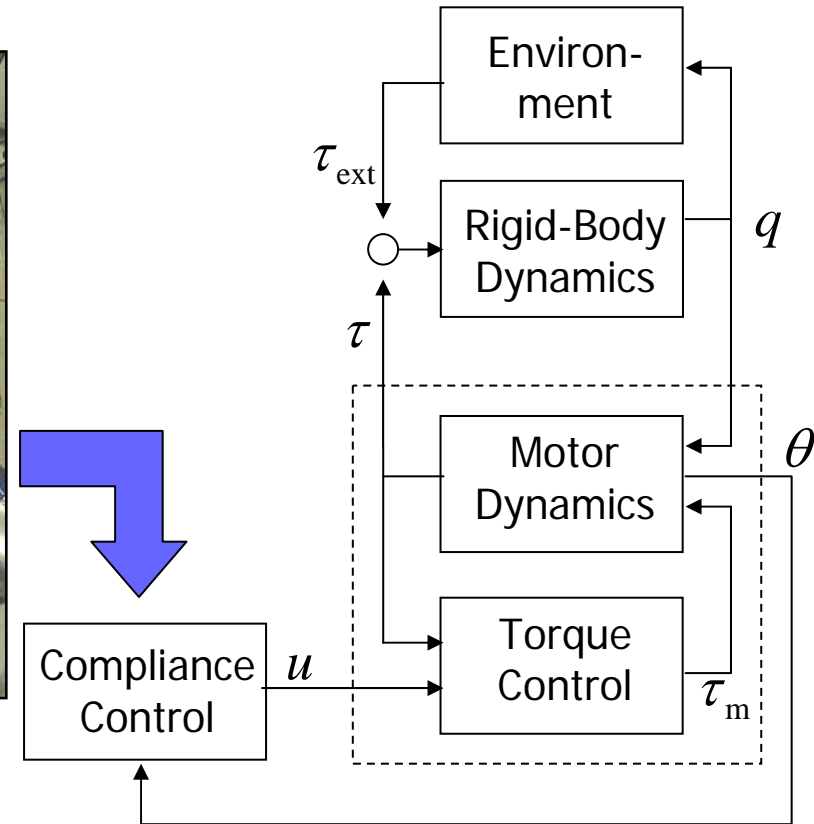
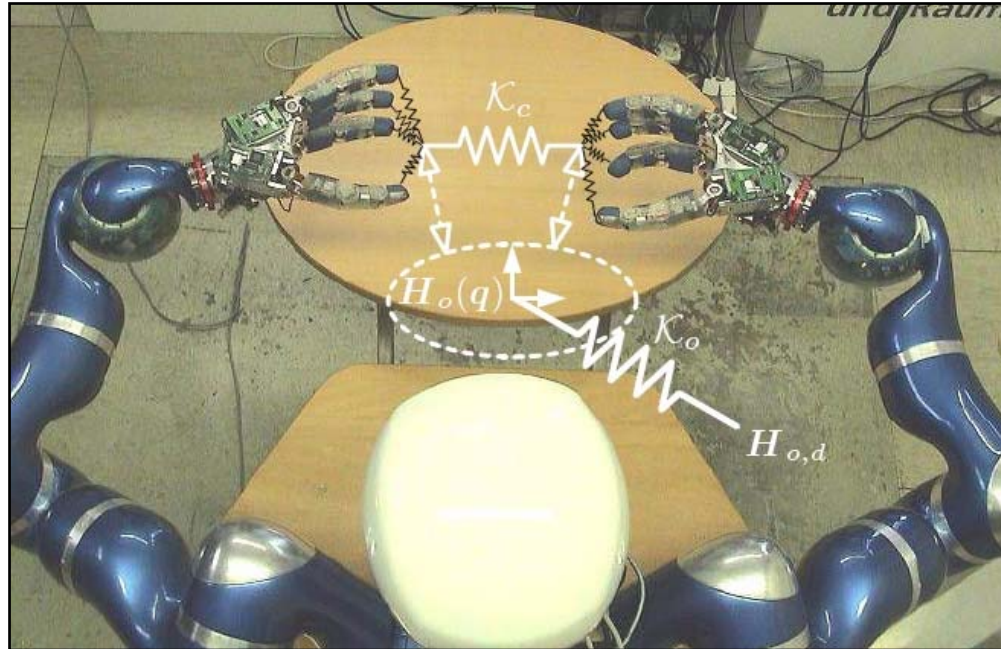
## Joint torque sensing & control for manipulation



**Robustness:**  
Passivity Based Control



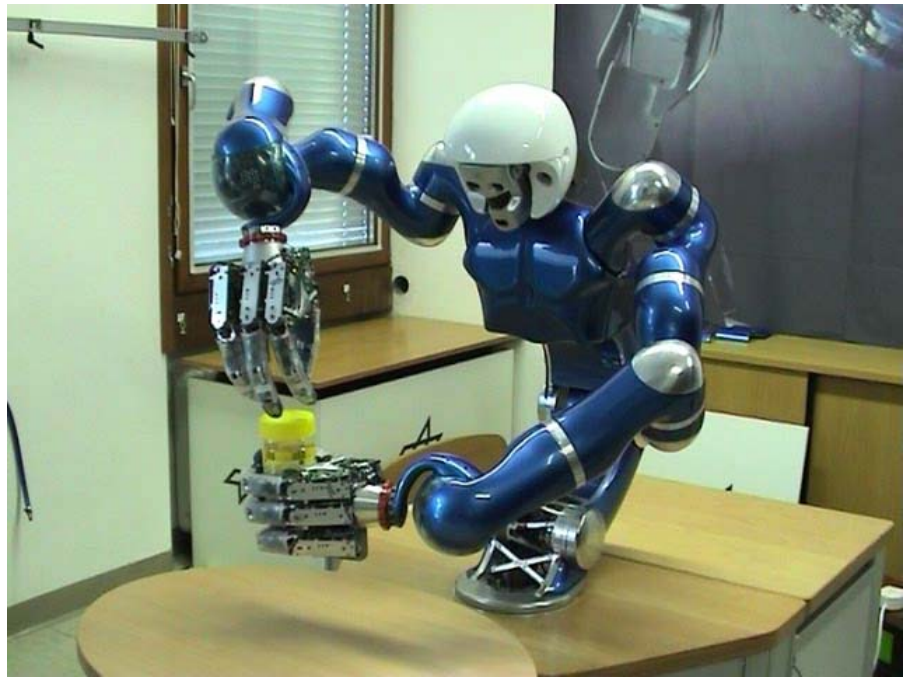
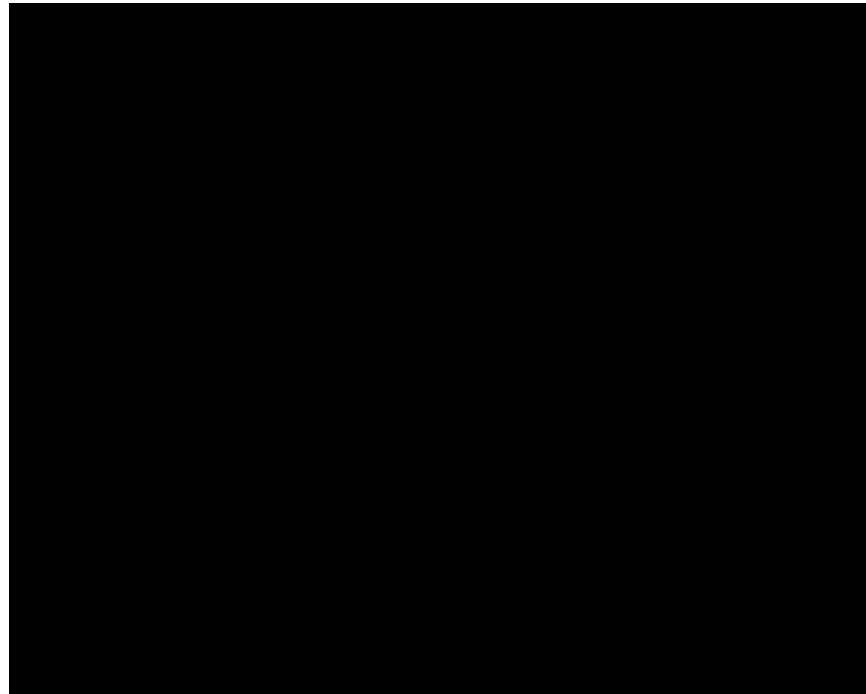
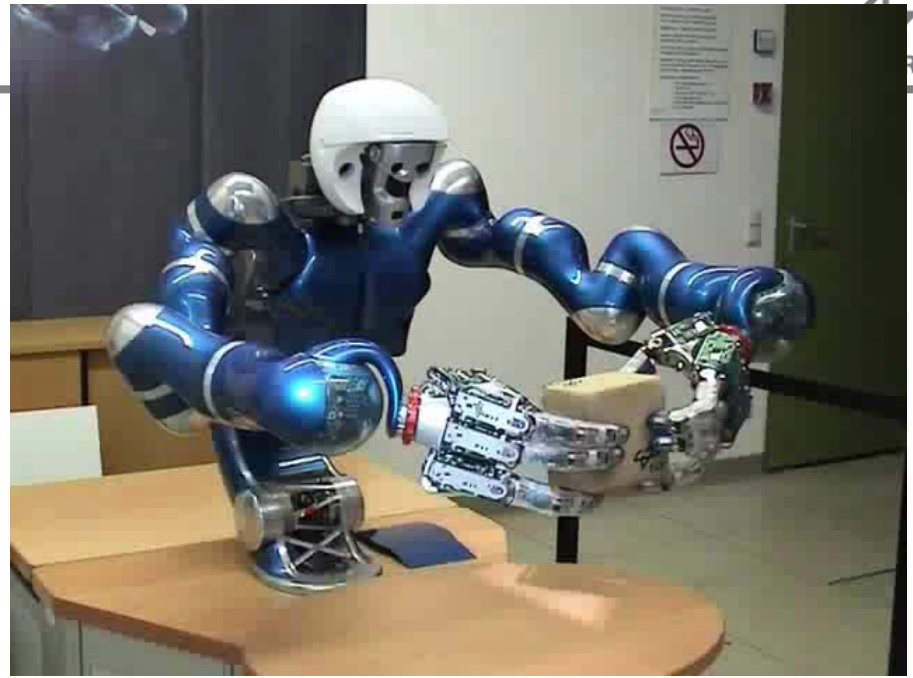
**Performance:**  
Joint Torque Feedback  
(noncollocated)



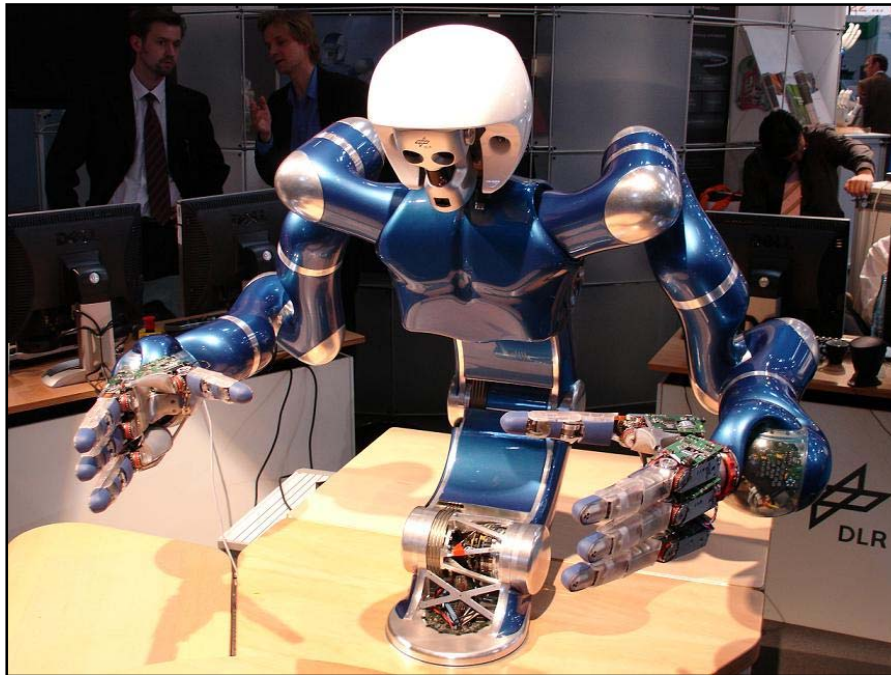
**Robustness:**  
Passivity Based Control



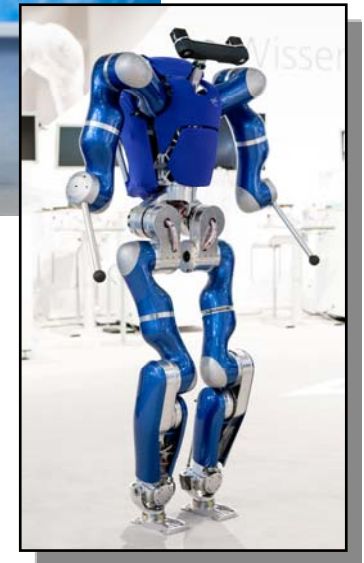
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Joint Torque Feedback  
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Joint torque sensing & control for manipulation

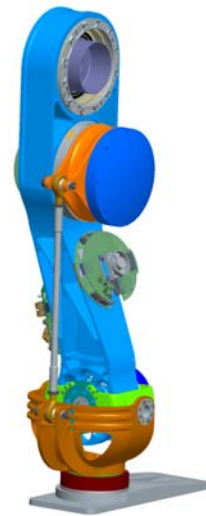


DLR-Biped [Humanoids 2010]



## Experimental biped walking machine [Humanoids 2010]

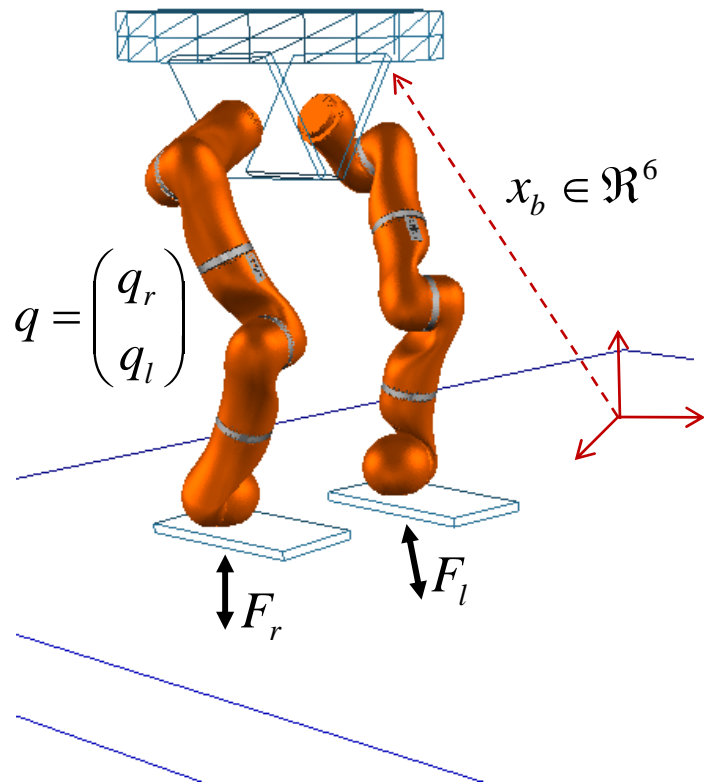
- 6 DOF / leg
- ~50 kg
- Drive technology of the DLR arm
- Newly designed lower leg
- Slim foot design: 19 x 9,5cm
- Sensors:
  - joint torque sensors
  - force/torque sensors in the feet
  - IMU in the trunk
- Developed within 10 month by student projects.
- Allow for position controlled walking (ZMP) and joint torque control!



## (TORO)

- Very recent development
  - 1) preliminary version: May 2012
  - 2) full version: December 2012 (estimated)
  
- Research interests:
  - Whole body motion/dynamics
  - Multi-contact interaction
  
- Weight: ~68kg / 75kg (complete)
  
- Modified hip kinematics:  
compact design for locating the  
total COM close to the hip joints





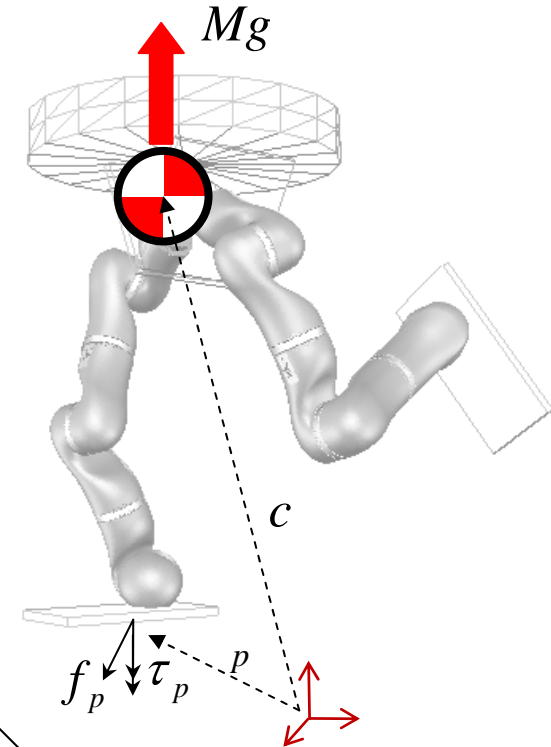
## Properties for control:

- Underactuated
- Varying unilateral constraints (single support, double support, edge contact)
- Constraints on the state & control

$$\begin{bmatrix} M_x(q) & M_{xq}(q) \\ M_{qx}(q) & M(q) \end{bmatrix} \begin{pmatrix} \ddot{x}_b \\ \ddot{q} \end{pmatrix} + \bar{C}(q, \dot{x}_b, \dot{q}) \begin{pmatrix} \dot{x}_b \\ \dot{q} \end{pmatrix} + \bar{g}(x_b, q) = \begin{pmatrix} 0 \\ \tau \end{pmatrix} + \begin{bmatrix} J_{br}(q)^T \\ J_r(q)^T \\ 0 \end{bmatrix} F_r + \begin{bmatrix} J_{bl}(q)^T \\ 0 \\ J_l(q)^T \end{bmatrix} F_l$$

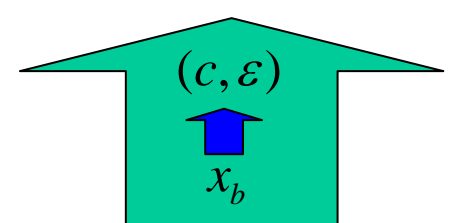


→ system structure with decoupled COM dynamics.  
[Space Robotics], [Wieber 2005, Hyon et al. 2006]



$$\begin{bmatrix} M & 0 \\ 0 & \hat{M}(q) \end{bmatrix} \begin{pmatrix} \ddot{c} \\ \ddot{\hat{q}} \end{pmatrix} + \begin{bmatrix} 0 \\ \hat{C}(\hat{q}, \dot{\hat{q}}) \end{bmatrix} + \begin{bmatrix} -Mg \\ 0 \end{bmatrix} = \begin{pmatrix} 0 \\ u \end{pmatrix} - \sum_{i=r,l} \begin{bmatrix} I & 0 \\ J_i & (\hat{q})^T \end{bmatrix} F_i$$

$\begin{pmatrix} \varepsilon \\ q \end{pmatrix}$ 
 $\begin{pmatrix} \tau_p \\ \tau \end{pmatrix}$



$$\begin{bmatrix} M_x(q) & M_{xq}(q) \\ M_{qx}(q) & M(q) \end{bmatrix} \begin{pmatrix} \ddot{x}_b \\ \ddot{q} \end{pmatrix} + \bar{C}(q, \dot{x}_b, \dot{q}) \begin{pmatrix} \dot{x}_b \\ \dot{q} \end{pmatrix} + \bar{g}(x_b, q) = \begin{pmatrix} 0 \\ \tau \end{pmatrix} + \begin{bmatrix} J_{br}(q)^T \\ J_r(q)^T \\ 0 \end{bmatrix} F_r + \begin{bmatrix} J_{bl}(q)^T \\ 0 \\ J_l(q)^T \end{bmatrix} F_l$$

On a flat ground:

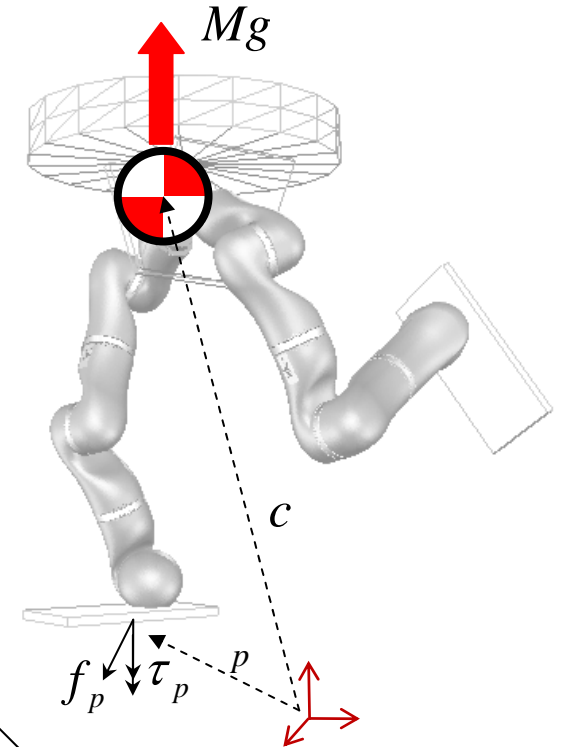
$$\tau_p = \dot{L} - c \times Mg - p \times (M\ddot{c} - Mg)$$

Conservation of angular momentum:

$$\dot{L} = c \times Mg + \sum \tau_i$$

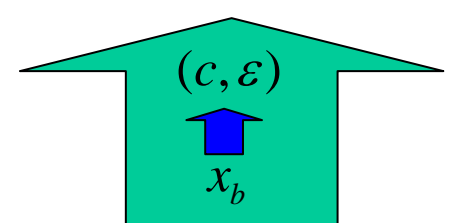
Conservation of momentum:

$$M\ddot{c} = Mg - \sum_{i=r,l} f_i$$



$$\begin{bmatrix} M & 0 \\ 0 & \hat{M}(q) \end{bmatrix} \begin{pmatrix} \ddot{c} \\ \hat{q} \end{pmatrix} + \begin{bmatrix} 0 \\ \hat{C}(\hat{q}, \dot{\hat{q}}) \end{bmatrix} + \begin{bmatrix} -Mg \\ 0 \end{bmatrix} = \begin{pmatrix} 0 \\ u \end{pmatrix} - \sum_{i=r,l} \begin{bmatrix} I & 0 \\ J_i(\hat{q})^T & \end{bmatrix} F_i$$

$$\begin{pmatrix} \varepsilon \\ q \end{pmatrix}$$

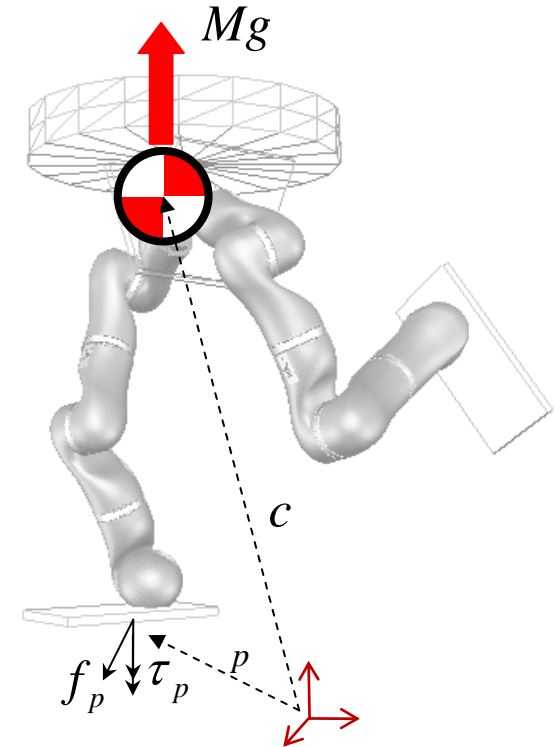


$$\begin{pmatrix} \tau_p \\ \tau \end{pmatrix}$$

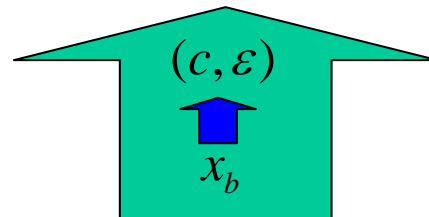
$$\begin{bmatrix} M_x(q) & M_{xq}(q) \\ M_{qx}(q) & M(q) \end{bmatrix} \begin{pmatrix} \ddot{x}_b \\ \ddot{q} \end{pmatrix} + \bar{C}(q, \dot{x}_b, \dot{q}) \begin{pmatrix} \dot{x}_b \\ \dot{q} \end{pmatrix} + \bar{g}(x_b, q) = \begin{pmatrix} 0 \\ \tau \end{pmatrix} + \begin{bmatrix} J_{br}(q)^T \\ J_r(q)^T \\ 0 \end{bmatrix} F_r + \begin{bmatrix} J_{bl}(q)^T \\ 0 \\ J_l(q)^T \end{bmatrix} F_l$$

On a flat ground: the center of pressure = ZMP

$$\tau_p = \dot{L} - c \times Mg - p \times (M\ddot{c} - Mg)$$

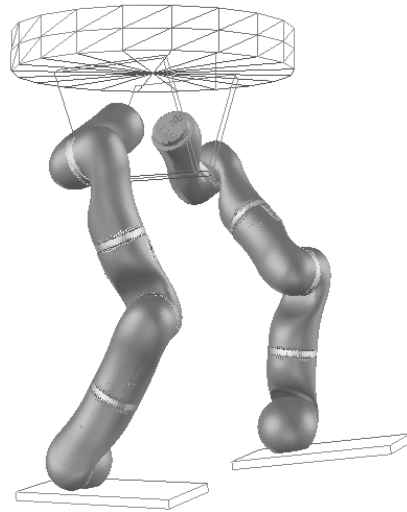


$$\begin{bmatrix} M & 0 \\ 0 & \hat{M}(q) \end{bmatrix} \begin{bmatrix} \ddot{c} \\ \ddot{\hat{q}} \end{bmatrix} + \begin{bmatrix} 0 \\ \hat{C}(\hat{q}, \dot{\hat{q}}) \end{bmatrix} + \begin{bmatrix} -Mg \\ 0 \end{bmatrix} = \begin{pmatrix} 0 \\ u \end{pmatrix} - \sum_{i=r,l} \begin{bmatrix} I & 0 \\ J_i & (\hat{q})^T \end{bmatrix} F_i$$

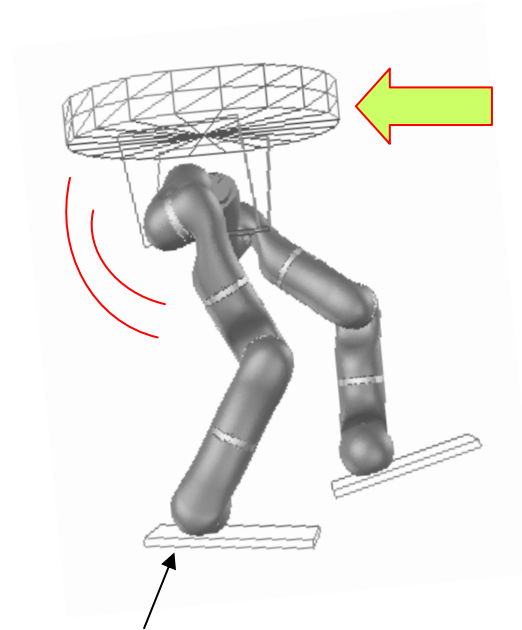


$$\begin{bmatrix} M_x(q) & M_{xq}(q) \\ M_{qx}(q) & M(q) \end{bmatrix} \begin{bmatrix} \ddot{x}_b \\ \ddot{q} \end{bmatrix} + \bar{C}(q, \dot{x}_b, \dot{q}) \begin{bmatrix} \dot{x}_b \\ \dot{q} \end{bmatrix} + \bar{g}(x_b, q) = \begin{pmatrix} 0 \\ \tau \end{pmatrix} + \overbrace{\begin{bmatrix} J_{br}(q)^T \\ J_r(q)^T \\ 0 \end{bmatrix} F_r + \begin{bmatrix} J_{bl}(q)^T \\ 0 \\ J_l(q)^T \end{bmatrix} F_l}$$

## Walking Control

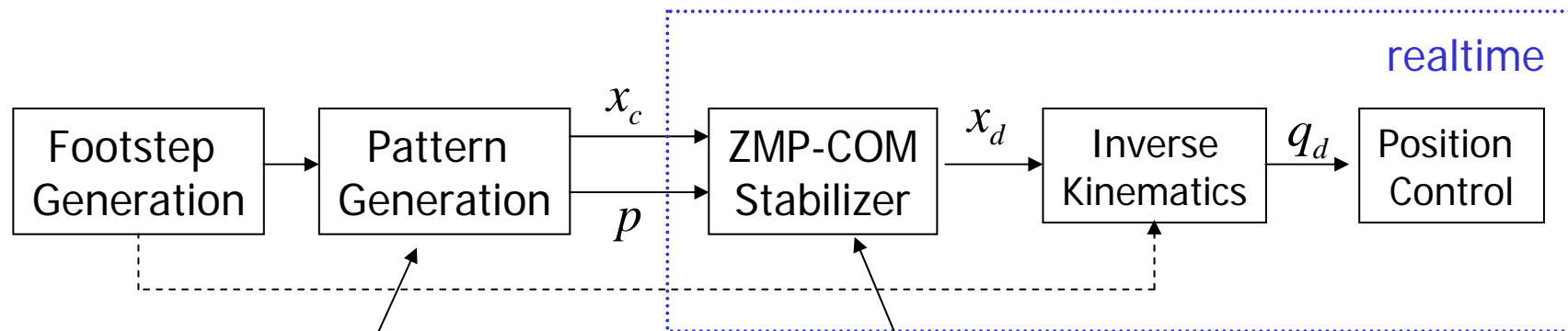


## Compliant Balancing



## State of the art walking control for fully actuated robots

1. Pattern Generator for desired CoM and ZMP motion
2. ZMP based Stabilizer



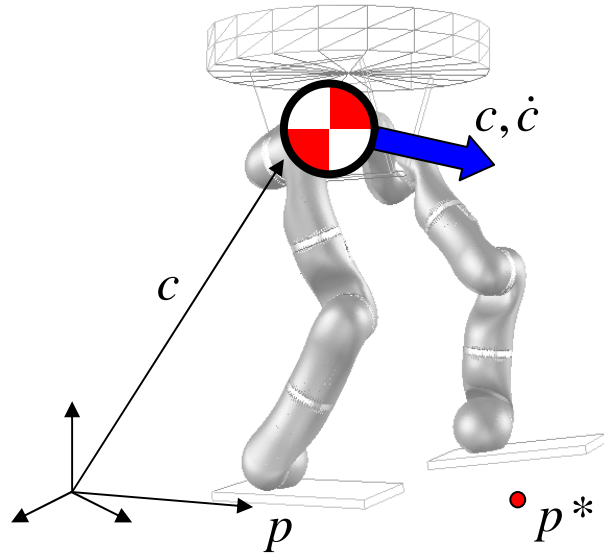
e.g. Preview Control [Kajita, 2003]

e.g. [Choi et al., 2007]

Model Predictive Control [Wieber, 2006]

Definition of the "Capture Point" (Pratt 2006, Hof 2008):

Point to step in order to bring the robot to stand.



$$\tau_p = \dot{L} - c \times Mg - p \times (M\ddot{c} - Mg)$$

$$\begin{aligned} L &\approx c \times M\dot{c} & \tau_{px} &= 0 \\ z &= \text{const} & \tau_{py} &= 0 \end{aligned}$$

$$\ddot{x} = \omega^2 (x - p) \quad \omega = \sqrt{\frac{g}{z}}$$

↑  
ZMP

Computation of the Capture Point:

$$p = \text{const} \longrightarrow x(t) = \cosh(\omega t)x(0) + \sinh(\omega t)\frac{\dot{x}(0)}{\omega} + (1 - \cosh(\omega t))p$$

$$x(t \rightarrow \infty) = p$$

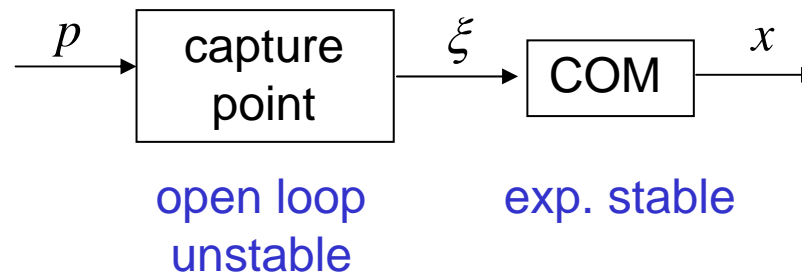
$$p^* = x_0 + \frac{\dot{x}_0}{\omega}$$

Coordinate transformation:  $(x, \dot{x}) \rightarrow (x, \xi)$

$$\xi = x + \frac{\dot{x}}{\omega}$$

$$\ddot{x} = \omega^2(x - p) \quad \longrightarrow \quad \begin{aligned} \dot{x} &= -\omega x + \omega \xi \\ \dot{\xi} &= \omega \xi - \omega p \end{aligned}$$

System structure: Cascaded system

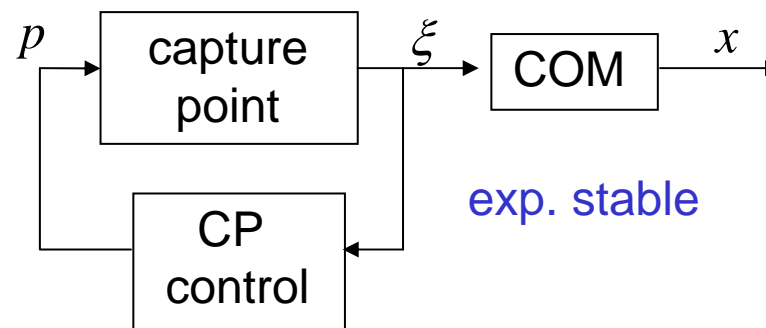


Coordinate transformation:  $(x, \dot{x}) \rightarrow (x, \xi)$

$$\xi = x + \frac{\dot{x}}{\omega}$$

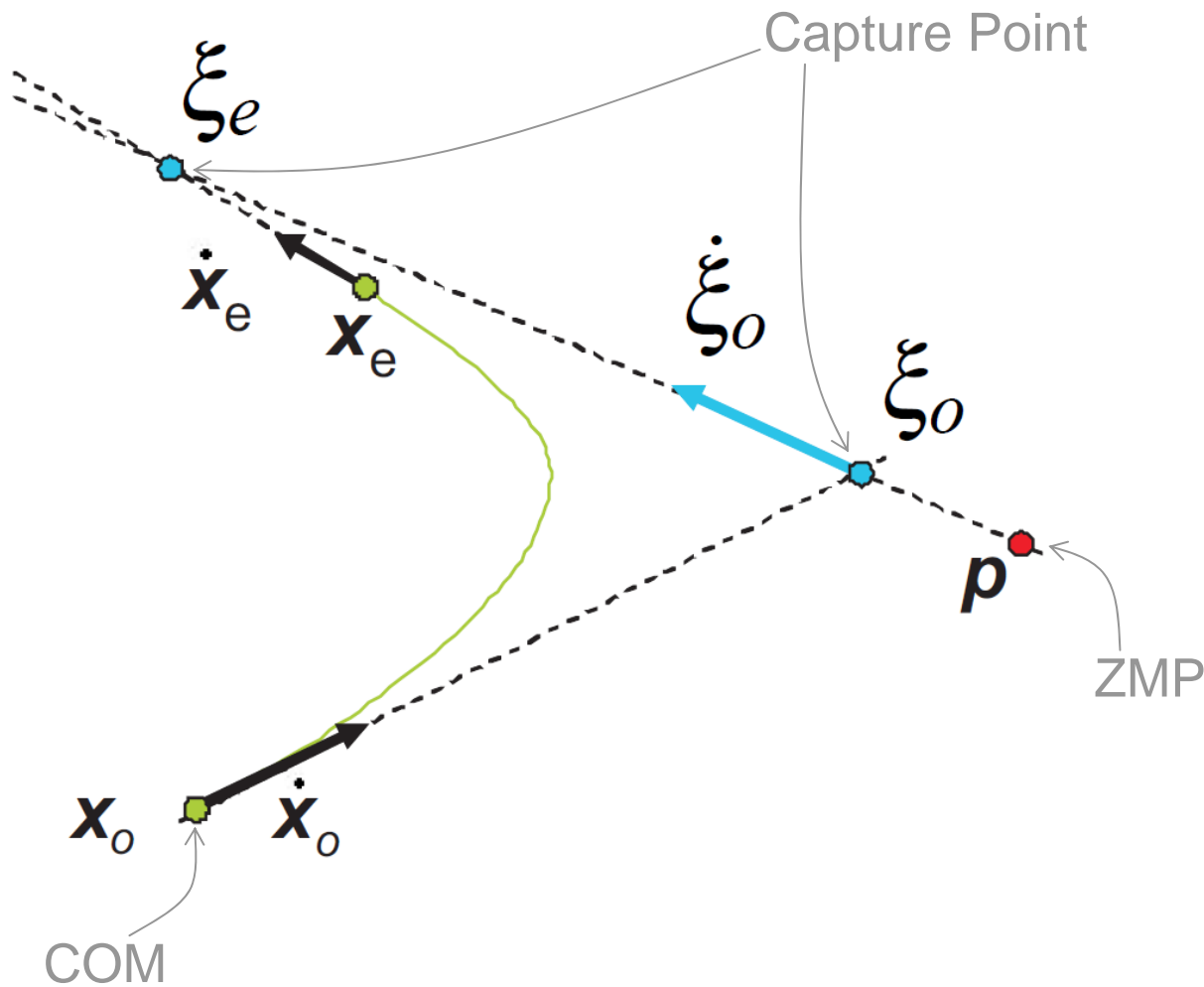
$$\ddot{x} = \omega^2(x - p) \quad \longrightarrow \quad \begin{aligned} \dot{x} &= -\omega x + \omega \xi \\ \dot{\xi} &= \omega \xi - \omega p \end{aligned}$$

System structure: Cascaded system



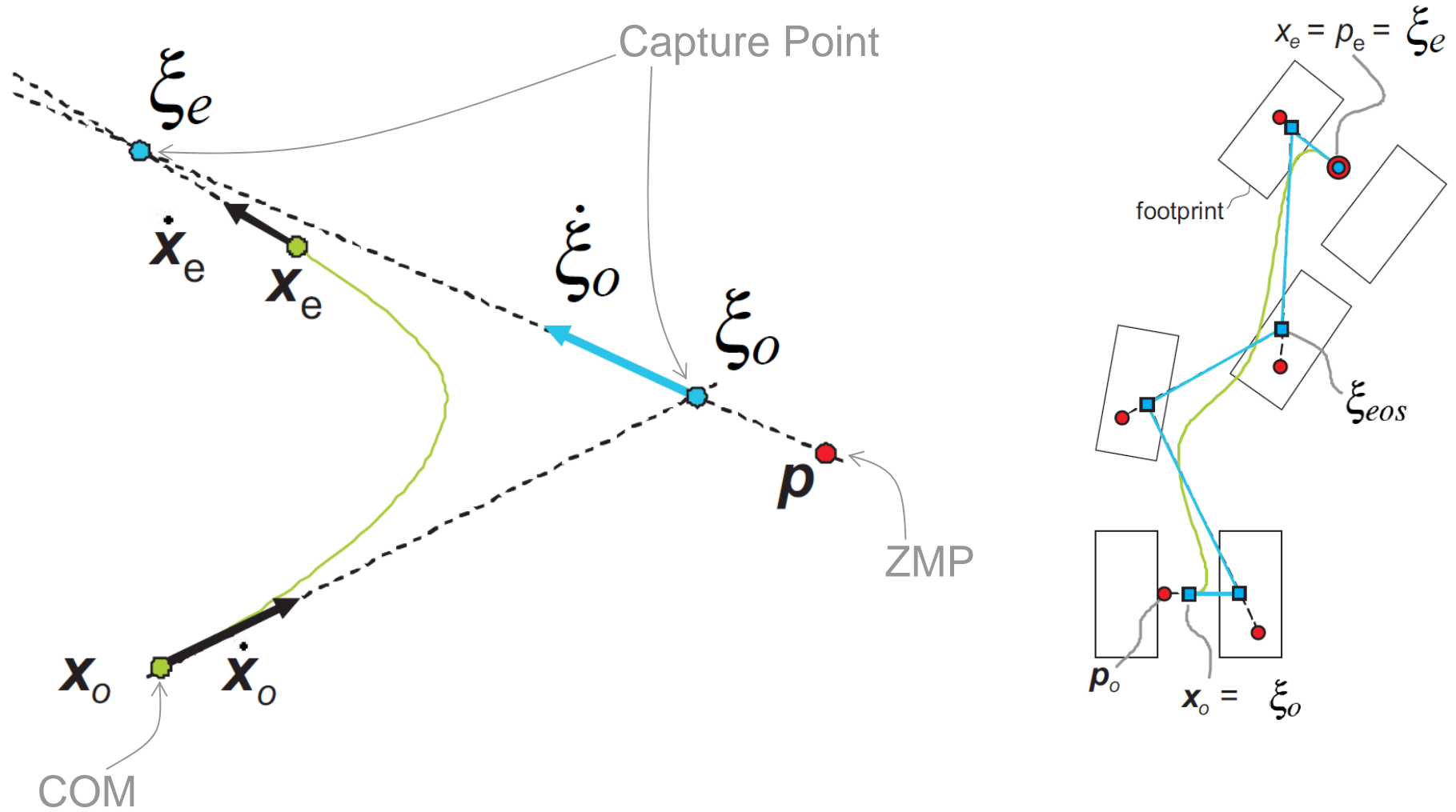
[Englsberger, Ott, et. al., IROS 2011]

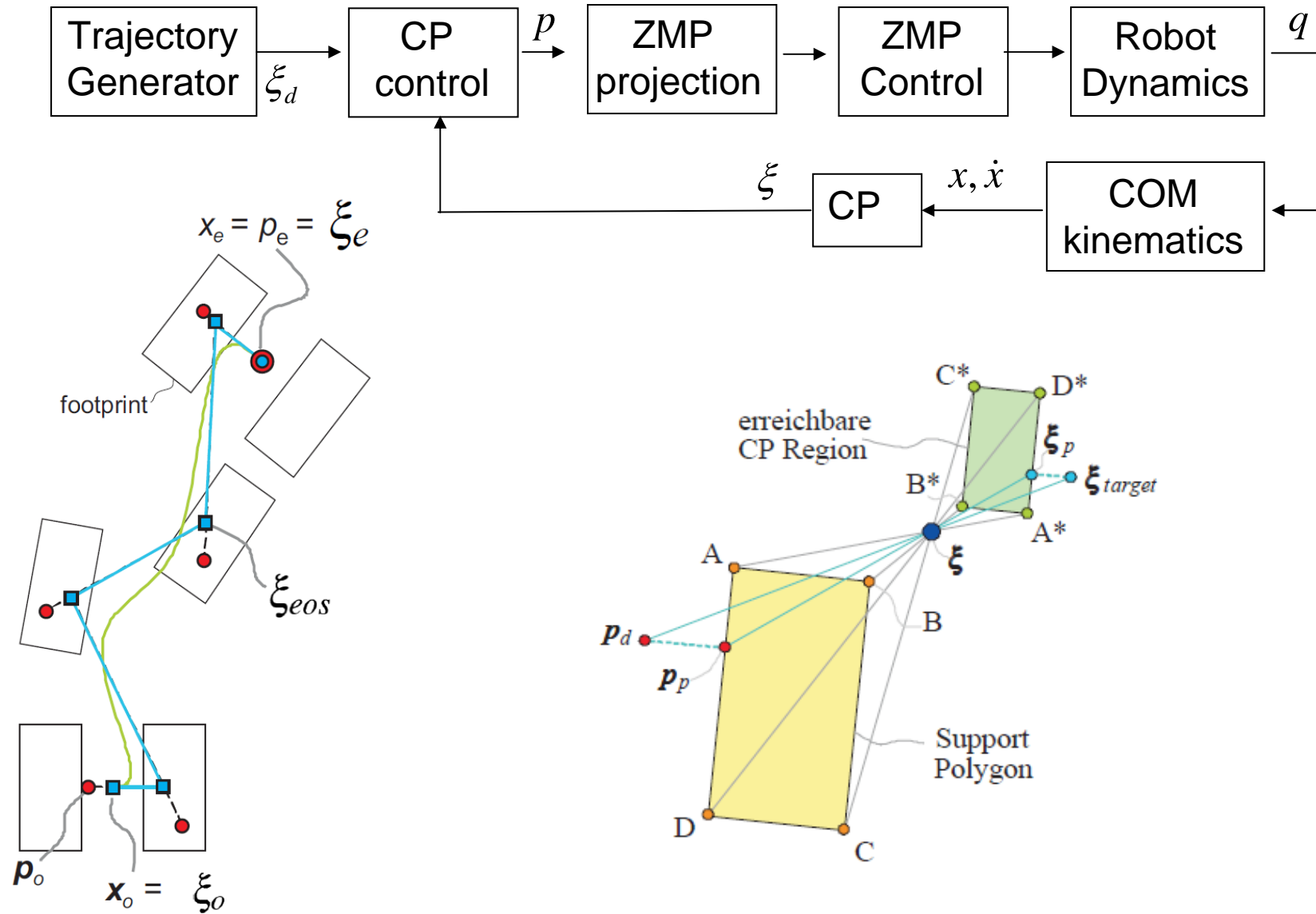


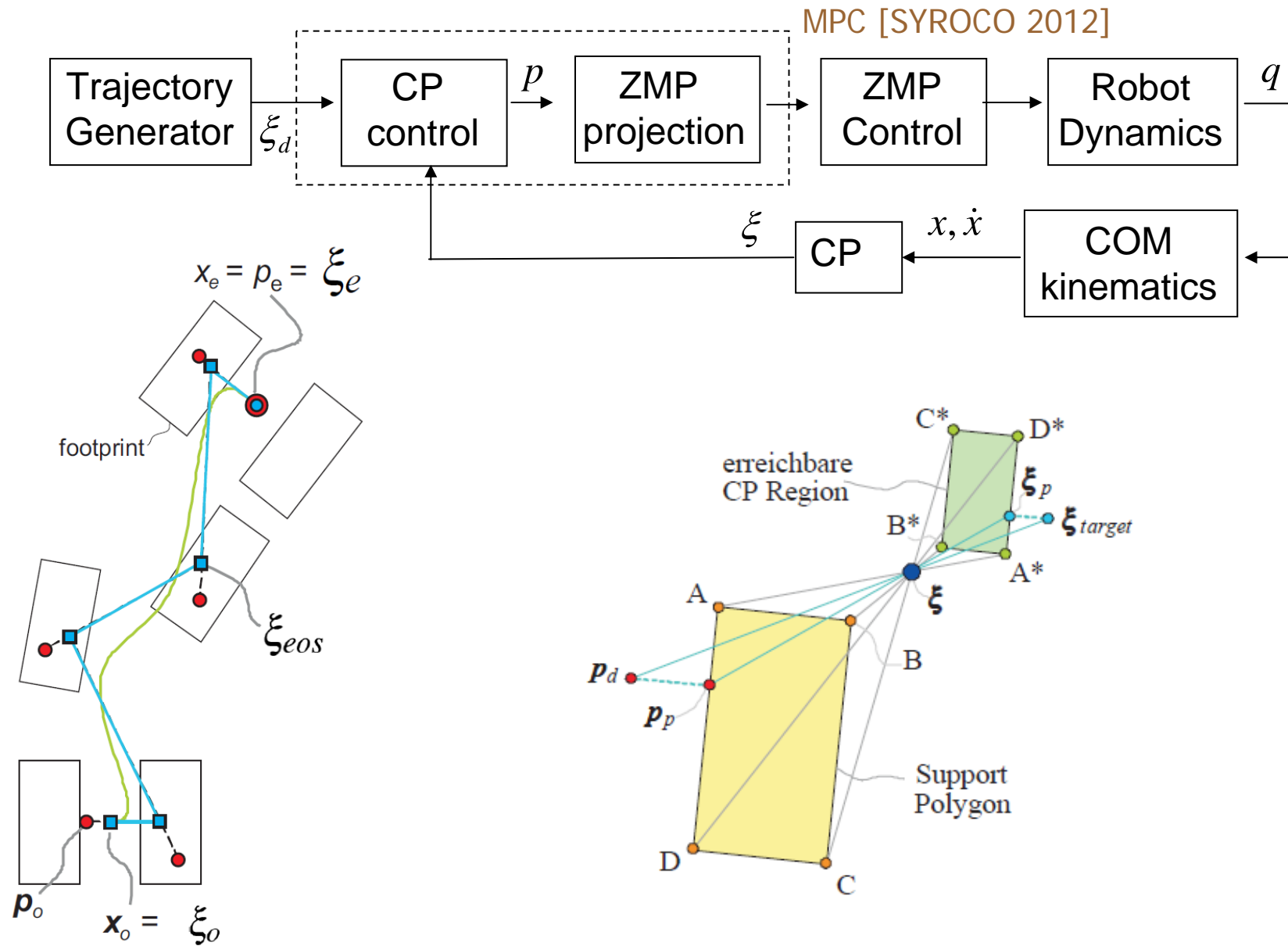


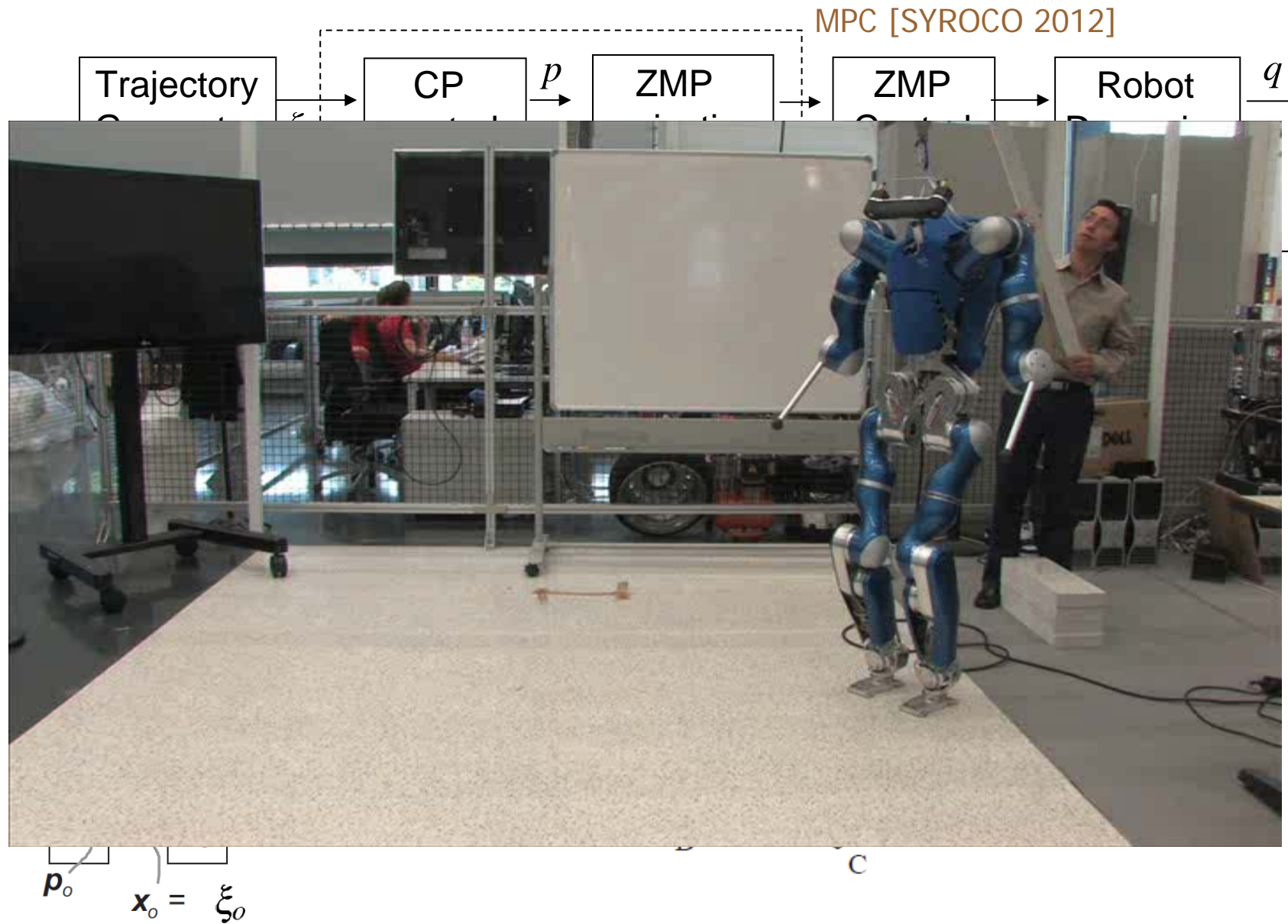
- COM velocity always points towards CP
- ZMP „pushes away“ the CP on a line
- COM follows CP

# Shifting the Capture Point





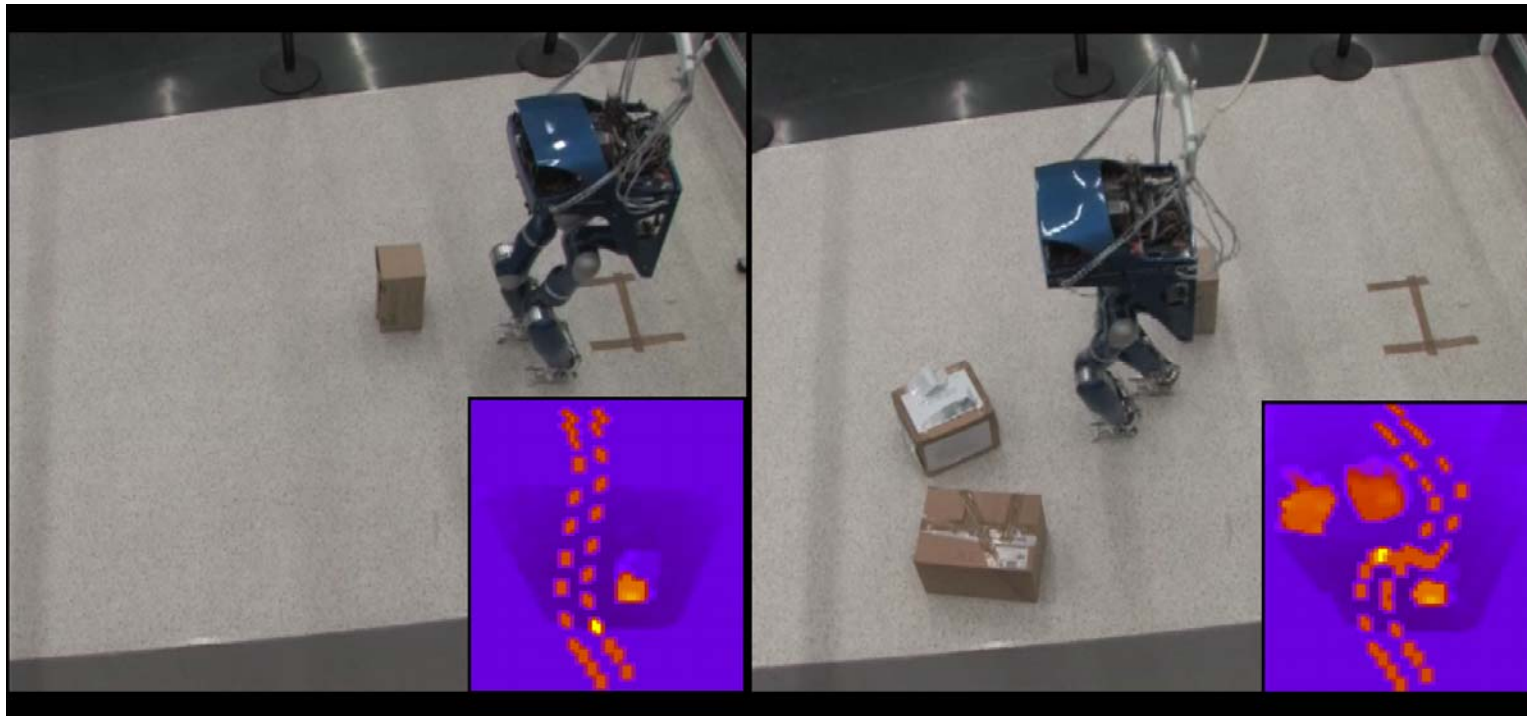




[Englsberger, Ott, et. al., IROS 2011]

## 1) Vision based walking

- stereo vision (Hirschmüller)
- visual SLAM (Chilian, Steidel)
- online footstep planning, collaboration with N. Perrin (IIT)





## 2) Optimized swingfoot trajectories: collaboration with H. Kaminaga (Univ. Tokyo)



- stride length: 70 cm
- speed: 0.5 m/s
- kinematically optimized swingfoot trajectory



- stride length: 70 cm
- speed: 0.5 m/s
- kinematically optimized torso motion  
(no angular momentum conversation! → slippery)

[Kaminaga et al., Humanoids 2012, Sat. Dec. 1st, 14:00]



## Simplified model

$$\ddot{x} = \omega^2(x - p)$$



$$\xi = x + \frac{\dot{x}}{\omega}$$

$$\begin{pmatrix} \dot{x} \\ \dot{\xi} \end{pmatrix} = \begin{bmatrix} -\omega & \omega \\ 0 & \omega \end{bmatrix} \begin{pmatrix} x \\ \xi \end{pmatrix} + \begin{bmatrix} 0 \\ -\omega \end{bmatrix} p$$

## General model

$$\ddot{x} = \frac{g + \ddot{z}}{z}(x - p) + \frac{\dot{L}}{Mz} \quad z = z(t)$$



$$\xi = x + \frac{\dot{x}}{\omega(t)} \quad \omega(t) = \sqrt{\frac{g}{z(t)}}$$

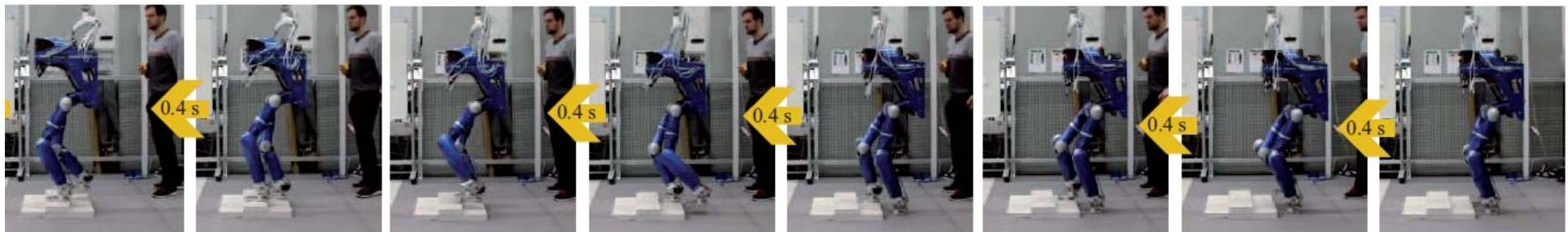
$$\begin{pmatrix} \dot{x} \\ \dot{\xi} \end{pmatrix} = \begin{bmatrix} -\omega(t) & \omega(t) \\ \frac{\ddot{z}}{\omega(t)z} - \frac{\dot{z}}{2z} & \omega(t) + \frac{\dot{z}}{2z} \end{bmatrix} \begin{pmatrix} x \\ \xi \end{pmatrix} + \begin{bmatrix} 0 \\ -\frac{g + \ddot{z}}{\omega(t)z} \end{bmatrix} p + \begin{bmatrix} 0 \\ \frac{\dot{L}}{M\omega(t)z} \end{bmatrix}$$

[Englsberger & Ott,  
Humanoids 2012]  
Poster I-19

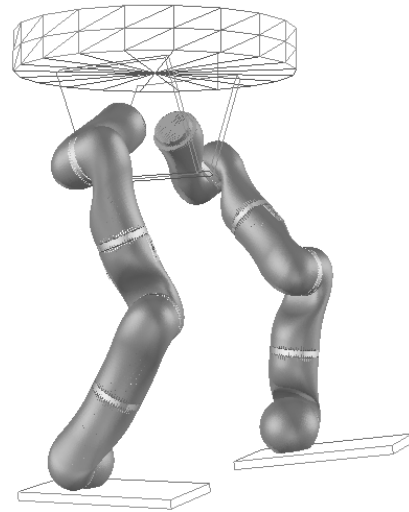


Feedback linearization  
→ timevarying cascaded  
dynamics

$$\begin{pmatrix} \dot{x} \\ \dot{\xi} \end{pmatrix} = \begin{bmatrix} -\omega(t) & \omega(t) \\ 0 & \omega(t) \end{bmatrix} \begin{pmatrix} x \\ \xi \end{pmatrix} + \begin{bmatrix} 0 \\ -\omega(t) \end{bmatrix} \hat{p}$$



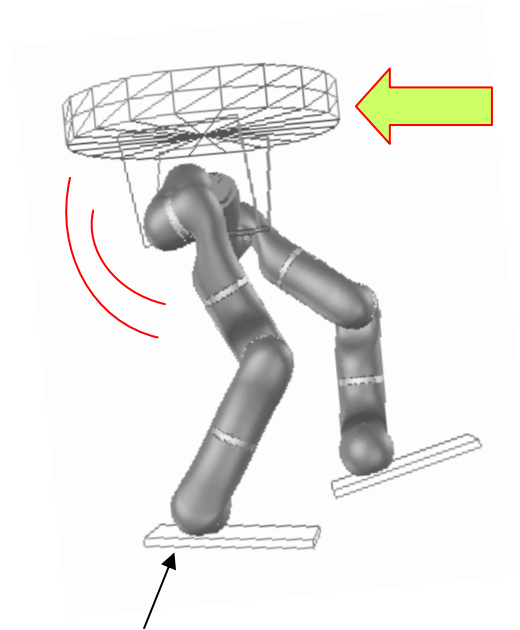
## Walking Control



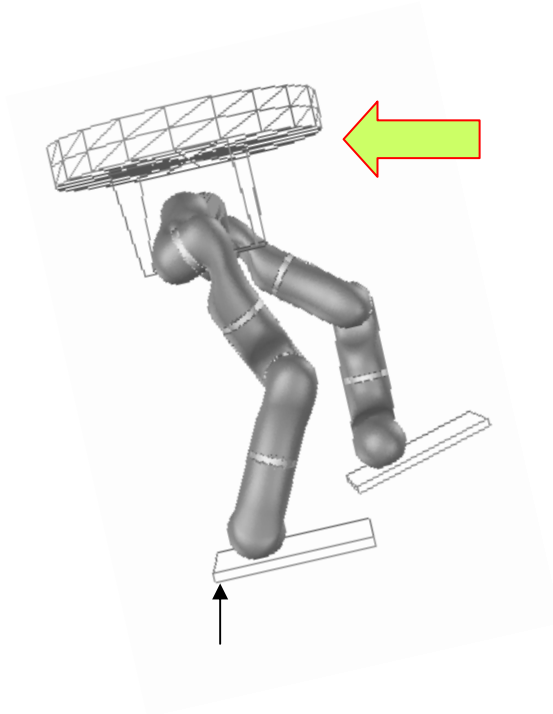
Use of the Capture Point

- ... simplifies control
- ... simplifies motion planning

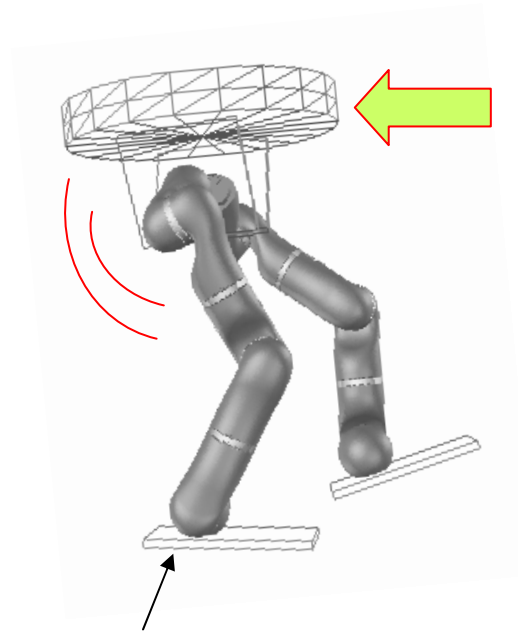
## Compliant Balancing



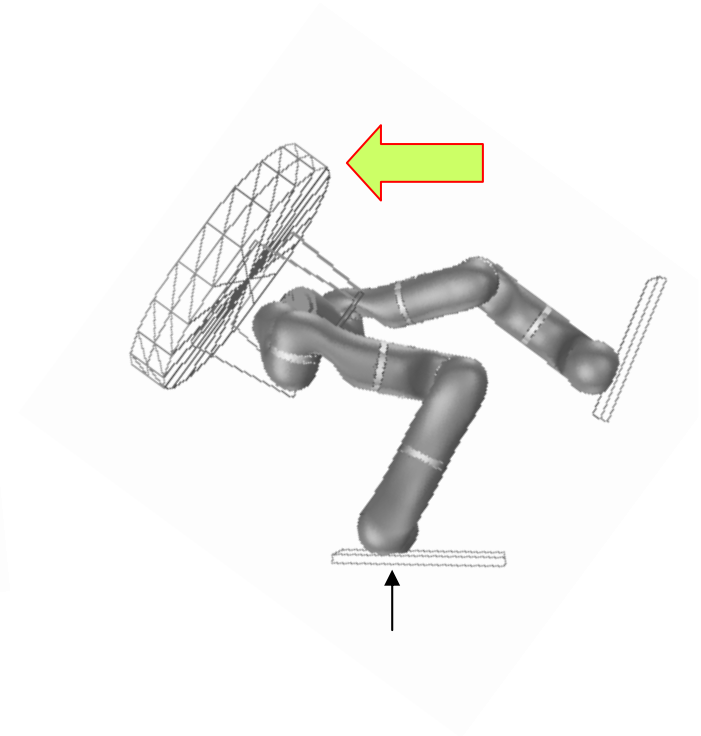
completely stiff



compliant control

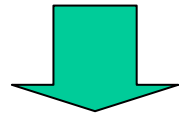


fully compliant

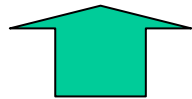


Compliant COM control [Hyon & Cheng, 2006]

$$F_{COM} = Mg - K_P(c - c_d) - K_D(\dot{c} - \dot{c}_d)$$



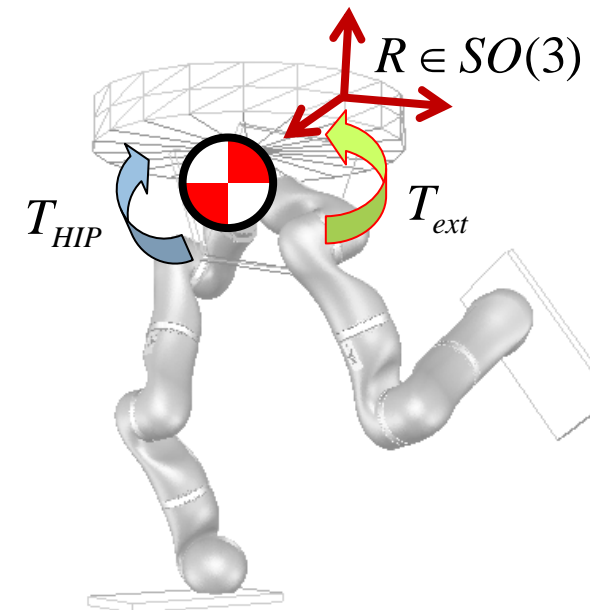
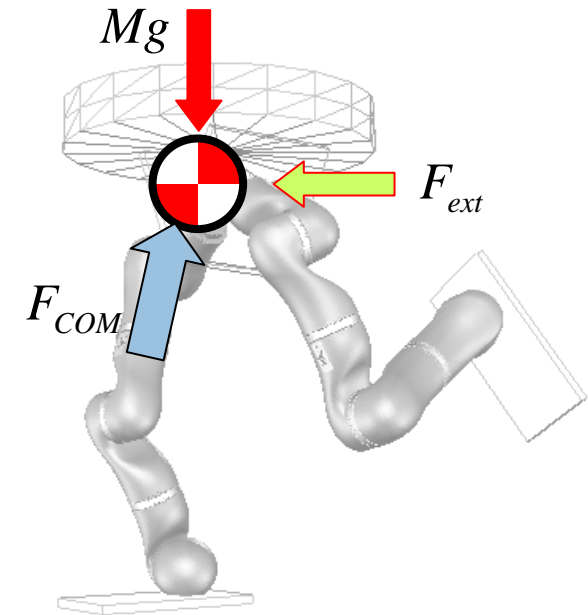
Desired wrench:  $W_d = (F_{COM}, T_{HIP})$



Trunk orientation Control

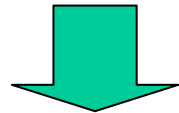
$$T_{HIP} = \frac{\partial \dot{V}(R, K_R)}{\partial \omega} + D_R(\omega - \omega_d)$$

IMU measurements

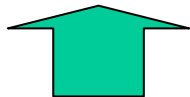


Compliant COM control [Hyon & Cheng, 2006]

$$F_{COM} = Mg - K_P(c - c_d) - K_D(\dot{c} - \dot{c}_d)$$



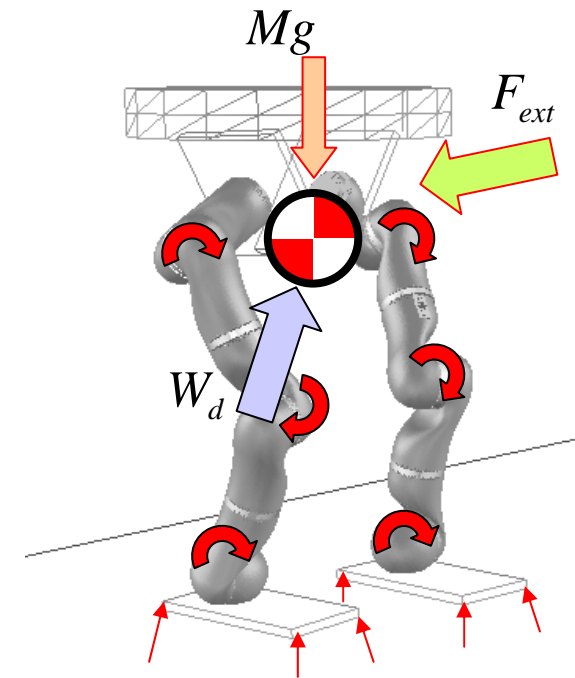
**Desired wrench:**  $W_d = (F_{COM}, T_{HIP})$



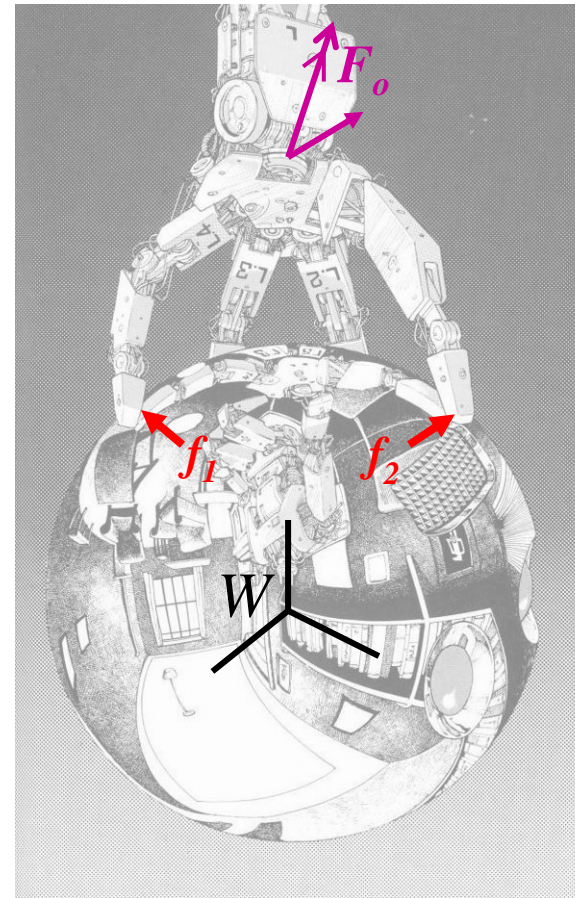
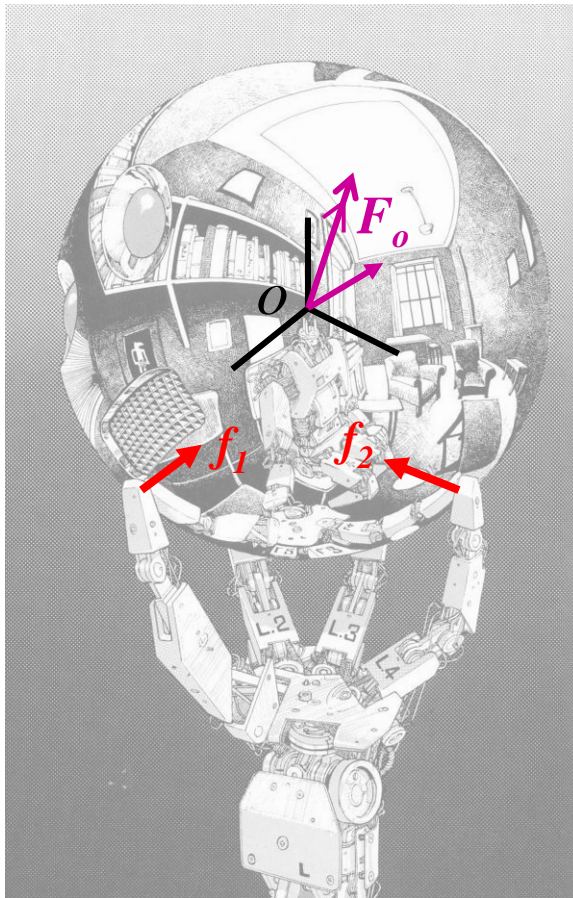
Trunk orientation Control

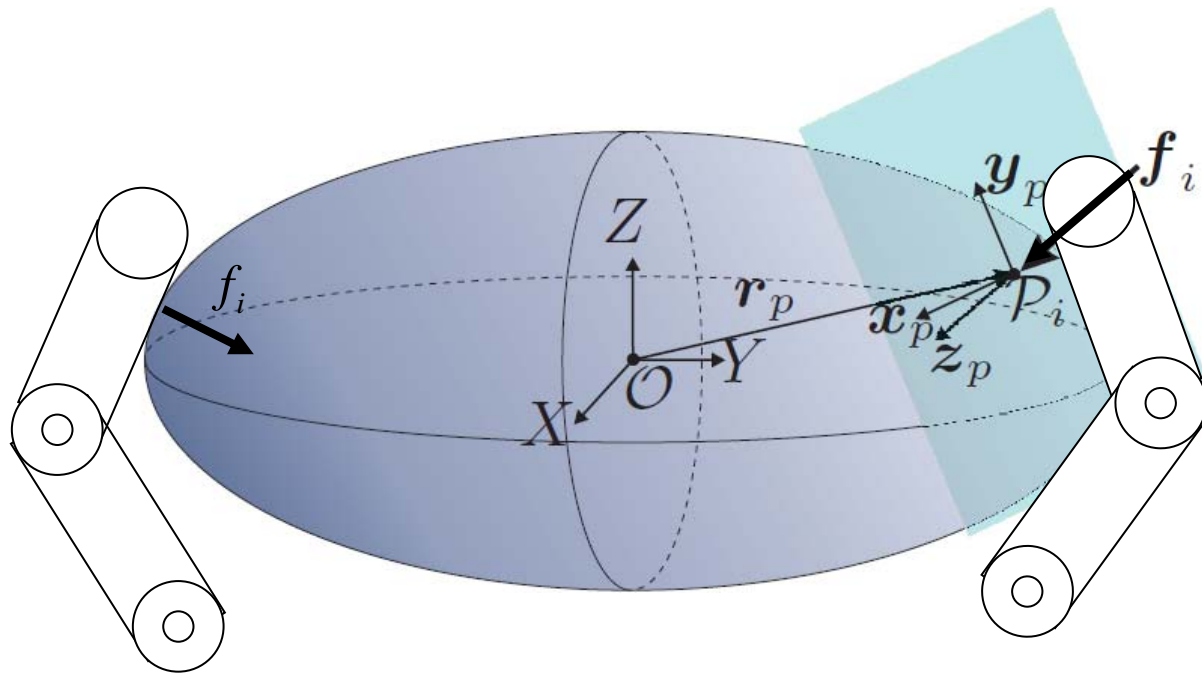
$$T_{HIP} = \frac{\partial \dot{V}(R, K_R)}{\partial \omega} + D_R(\omega - \omega_d)$$

IMU measurements



➤ Force distribution: Similar problems!





Net wrench acting on the object:

$$\underbrace{W_O}_{se(3)} = G_1 F_1 + \dots + G_\eta F_\eta = \underbrace{[G_1 \dots G_\eta]}_{\text{Grasp Map}} \underbrace{\begin{pmatrix} F_1 \\ \vdots \\ F_\eta \end{pmatrix}}_{F_C \in se(3)^\eta}$$

$$G_i = Ad_{P_i O}^T$$

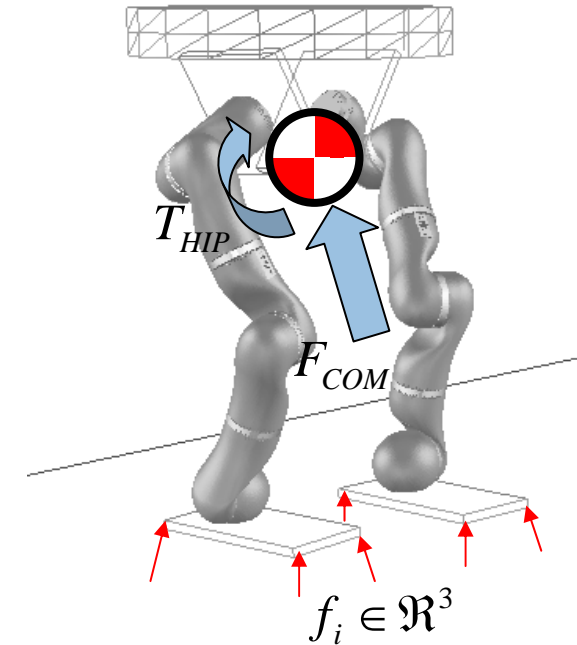
Well studied problem in grasping: Find contact wrenches  $F_C \in FC^\eta$  such that a desired net wrench on the object is achieved.

friction cone

## Relation between balancing wrench & contact forces

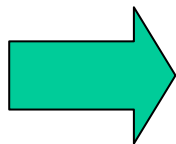
$$G_i = \begin{bmatrix} R_i \\ \hat{p}_i R_i \end{bmatrix}$$

$$W_d = \begin{bmatrix} G_1 & \dots & G_\eta \\ G_F \\ G_T \end{bmatrix} \begin{bmatrix} f_1 \\ \vdots \\ f_\eta \\ f_C \end{bmatrix}$$



### Constraints:

- Unilateral contact:  $f_{i,z} > 0$  (implicit handling of ZMP constraints)
- Friction cone constraints



Formulation as a constraint optimization problem

$$f_C = \arg \min \left\{ \alpha_1 \|F_{COM} - G_F f_C\|^2 + \alpha_2 \|T_{HIP} - G_T f_C\|^2 + \alpha_3 \|f_C\|^2 \right\} \quad \alpha_1 \gg \alpha_2 \gg \alpha_3$$



Multibody robot model:

COM as a base coordinate  $\rightarrow$  system structure with decoupled COM dynamics.

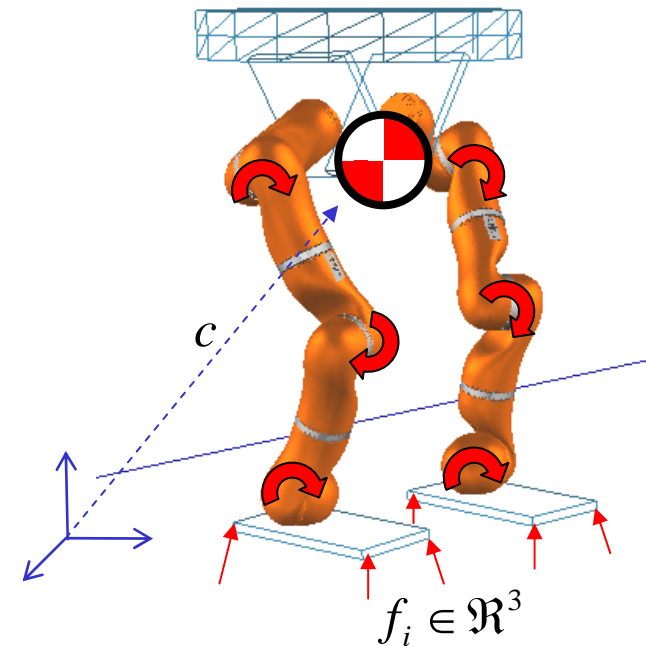
[Space Robotics], [Wieber 2005, Hyon et al. 2006]

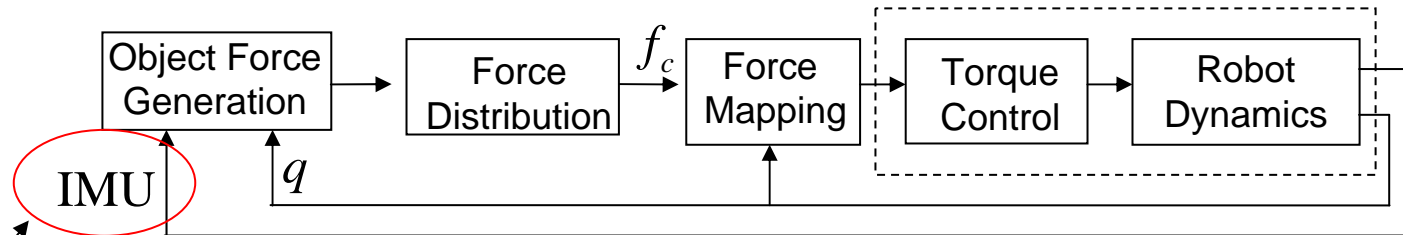
$$\begin{bmatrix} M & 0 \\ 0 & \hat{M}(q) \end{bmatrix} \begin{pmatrix} \ddot{c} \\ \ddot{\hat{q}} \end{pmatrix} + \begin{bmatrix} 0 \\ \hat{C}(\hat{q}, \dot{\hat{q}}) \end{bmatrix} + \begin{bmatrix} -Mg \\ 0 \end{bmatrix} = \begin{pmatrix} 0 \\ u \end{pmatrix} - \sum_{i=r,l} \begin{bmatrix} I & 0 \\ J_i(\hat{q})^T & \end{bmatrix} F_i$$

$M \ddot{c} = Mg - \sum f_i$

$$\tau = \sum J_i(\hat{q})^T f_i$$

Passivity based compliance control  
(well suited for balancing)





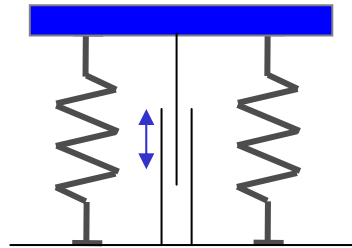
for orientation control and COM computation



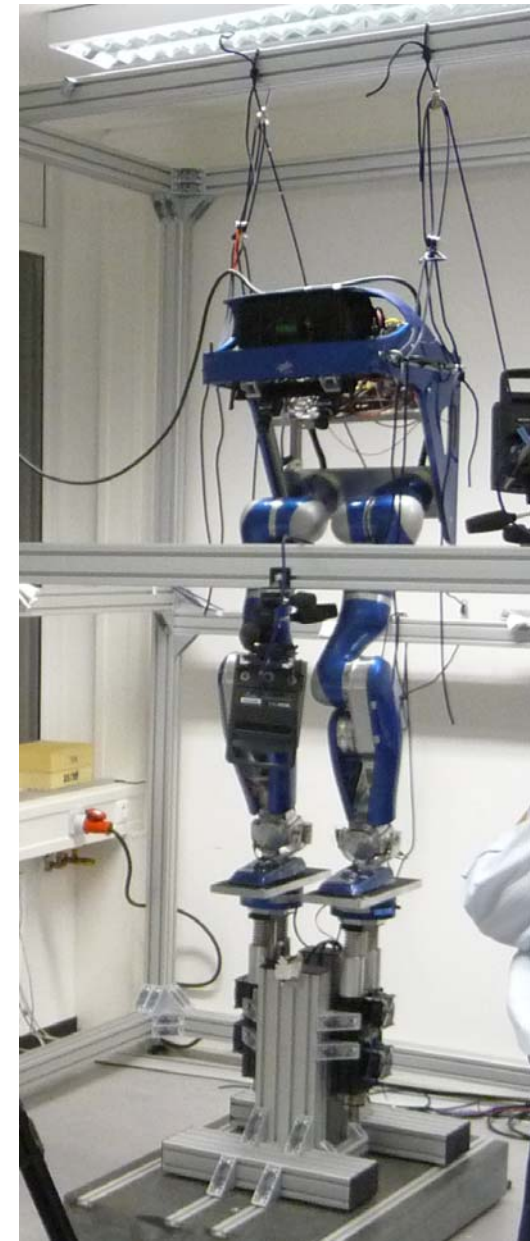


[Ott, Roa, Humanoids 2011, best paper award]

- Leg perturbation setup
- Movable elastic platform



- Experimental evaluation of the robustness with respect to disturbances (frequency & amplitude) at the foot

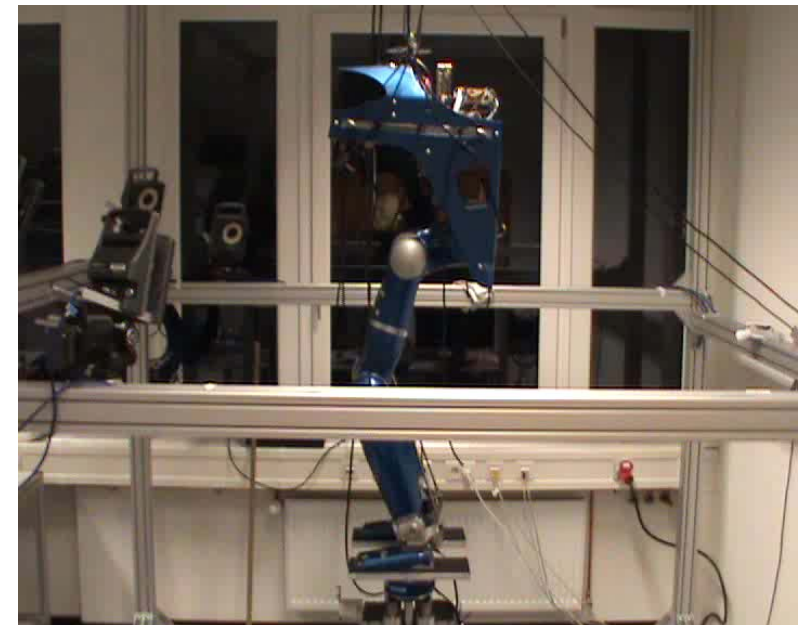
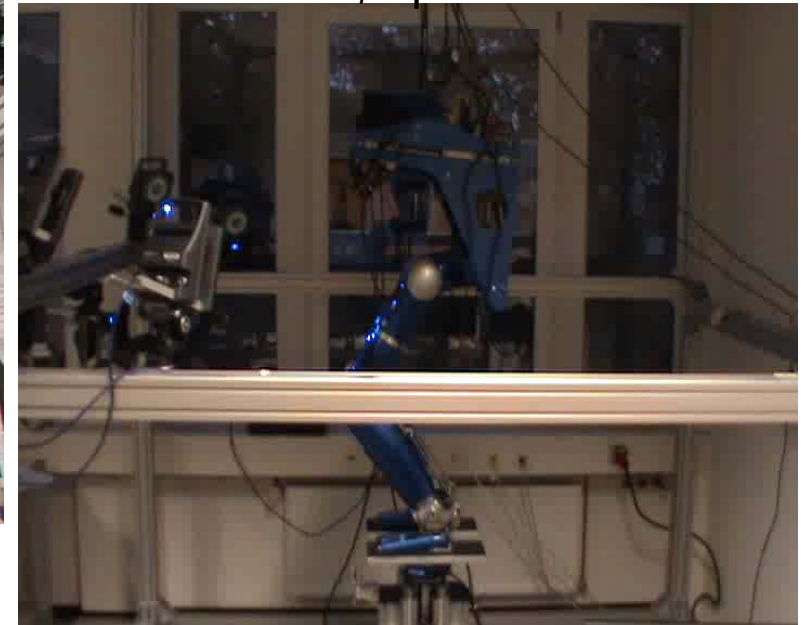




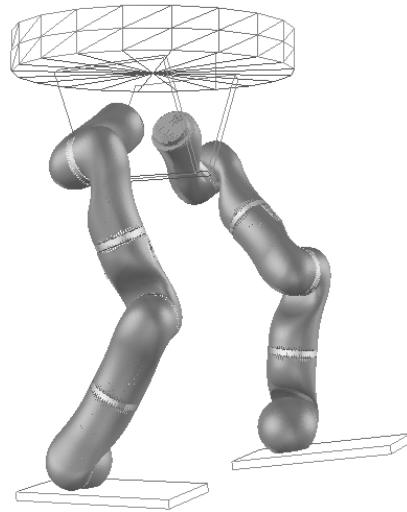
Out of phase disturbance



synchronous disturbance  
2mm, up to 8 Hz



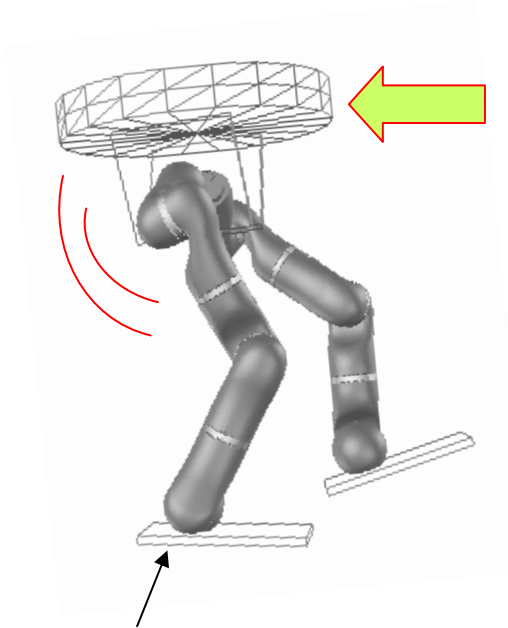
## Walking Control



Use of the Capture Point

- ... simplifies control
- ... simplifies motion planning

## Compliant Balancing



Joint torque sensing

- ... enables compliant control independently from precise foot-ground contact information.

- Compliance control for elastic robots based on joint torque sensing
- Walking control based on the Capture Point
- Extension of torque based compliance control to lower body balancing

## Outlook

- Combination of torque based balancing and CP based walking
  - realize robust walking on uneven terrain
- Multi-contact interaction using articulated upper body



# Thank you very much for your attention!

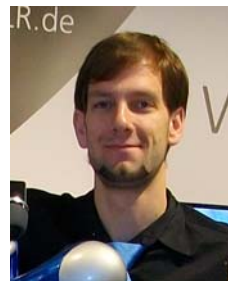
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