Walking motion Control: theory and implementation

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INRIA Grenoble

Who am I ?

- Master thesis on biped robots in 1997
- PhD thesis on biped robots in 2000
- My algorithms implemented in HRP-2, Nao and tested on other robots
- Never participated to any RoboCup

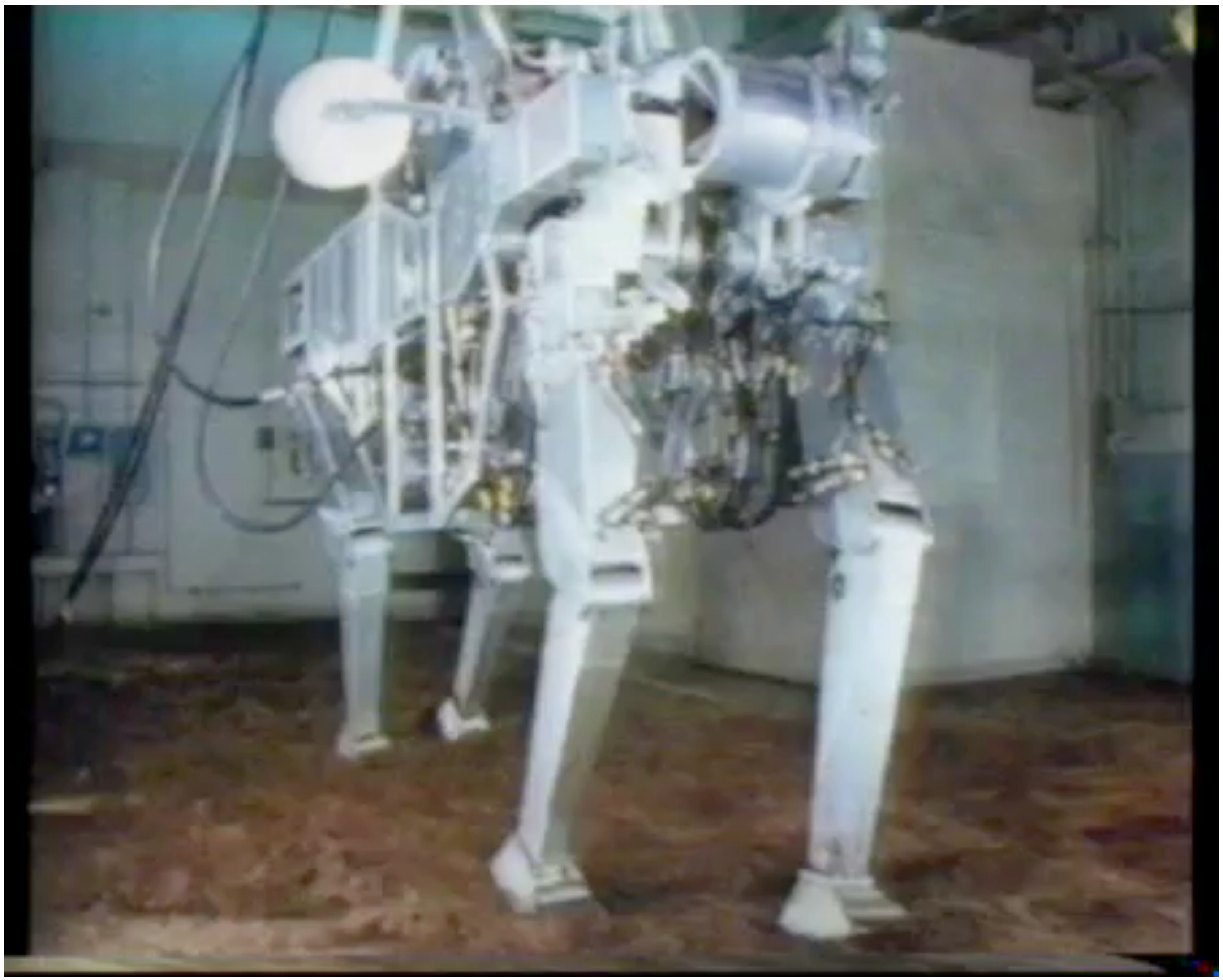
What are we going to see today ?

- Dynamics of legged locomotion
- Generation of dynamic walking motions
- Motion and force control
- Numerical implementation details

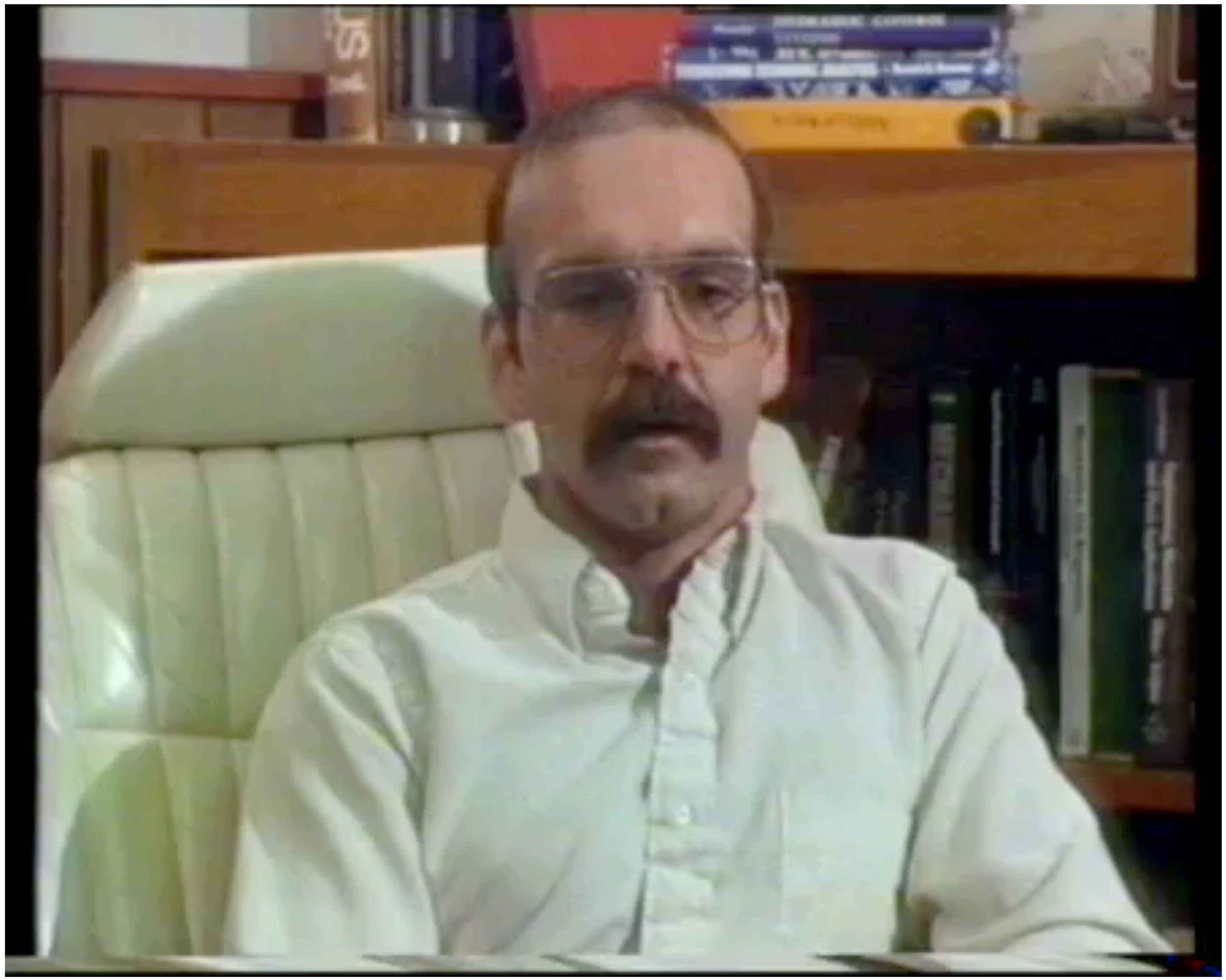
Milestones in legged robotics

- 1960s : Walking Truck (R. Mosher)
- 1970s : Waseda university (I. Kato)
- 1980s : Adaptive Suspension Vehicle (R. McGhee)
- 1980s : MIT LegLab (M. Raibert)
- 1996 : Honda P2 (K. Hirai, T. Takenaka...)

The walking truck



The adaptive suspension vehicle



MIT LegLab

The Honda P2

The dynamics of legged locomotion

Structure of the minimal coordinates

- Joint positions
- Position and orientation with respect to the environment

$$q = \begin{bmatrix} \hat{q} \\ x_0 \\ \theta_0 \end{bmatrix}$$

Structure of the Lagrangian dynamics

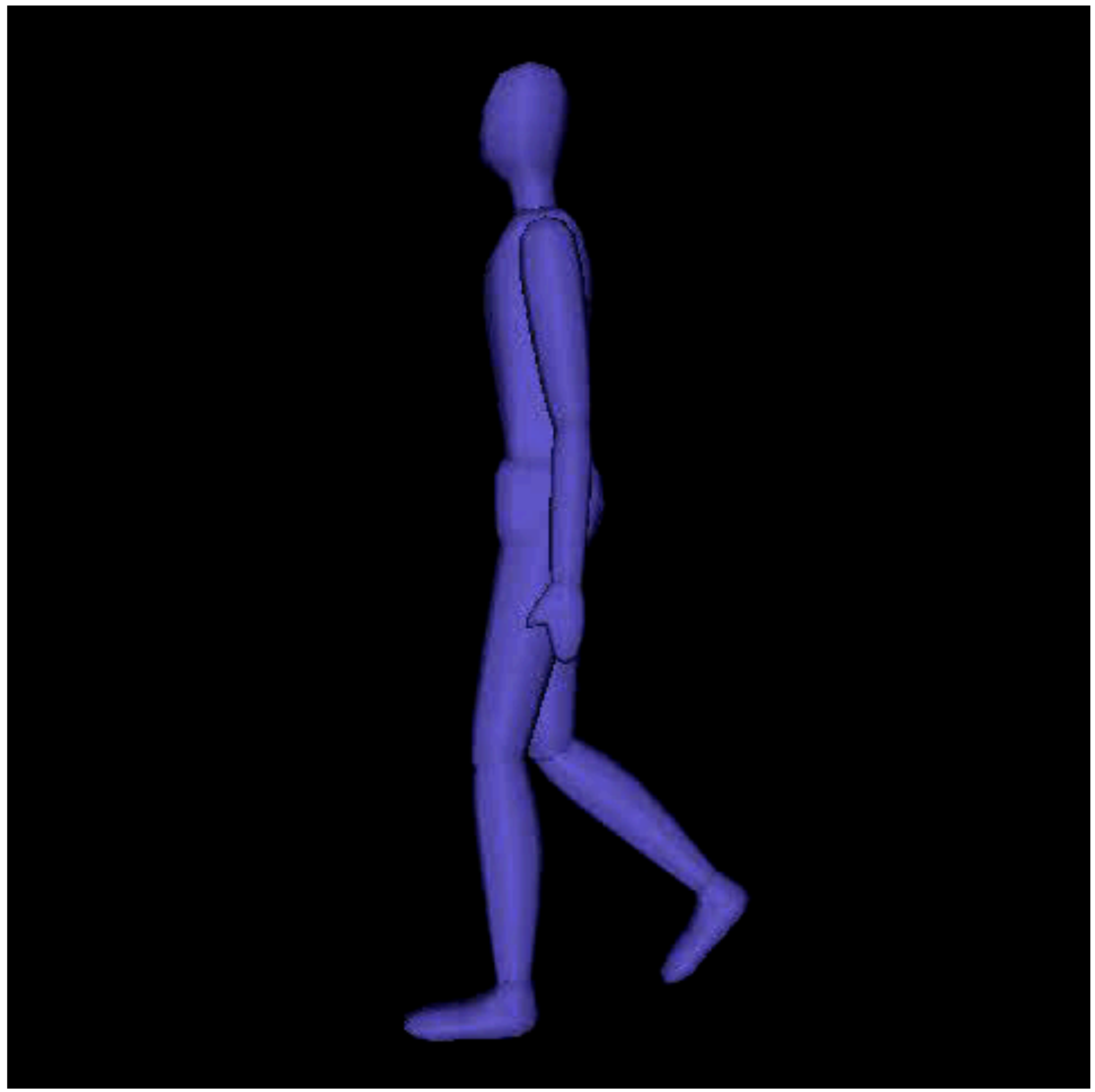
$$M(q)\left(\begin{bmatrix}\ddot{\hat{q}}\\ \ddot{x}_0\\ \ddot{\theta}_0\end{bmatrix}+\begin{bmatrix}0\\ g\\ 0\end{bmatrix}\right)+n(q,\dot{q})=\begin{bmatrix}u\\ 0\\ 0\end{bmatrix}+\sum_i C_i(q)^T f_i$$

$$m\left(\ddot{c}+g\right)=\sum_i f_i$$

$$\dot{L}=\sum_i (p_i-c)\times f_i$$

$$L=\sum_k (x_k-c)\times m_k\dot{x}_k+I_k\omega_k$$

And yet it moves



< 1 degree/step

Structure of the Lagrangian dynamics

$$M(q)\left(\begin{bmatrix}\ddot{\hat{q}}\\ \ddot{x}_0\\ \ddot{\theta}_0\end{bmatrix}+\begin{bmatrix}0\\ g\\ 0\end{bmatrix}\right)+n(q,\dot{q})=\begin{bmatrix}u\\ 0\\ 0\end{bmatrix}+\sum_i C_i(q)^T f_i$$

$$m\left(\ddot{c}+g\right)=\sum_i f_i$$

$$\dot{L}=\sum_i (p_i-c)\times f_i$$

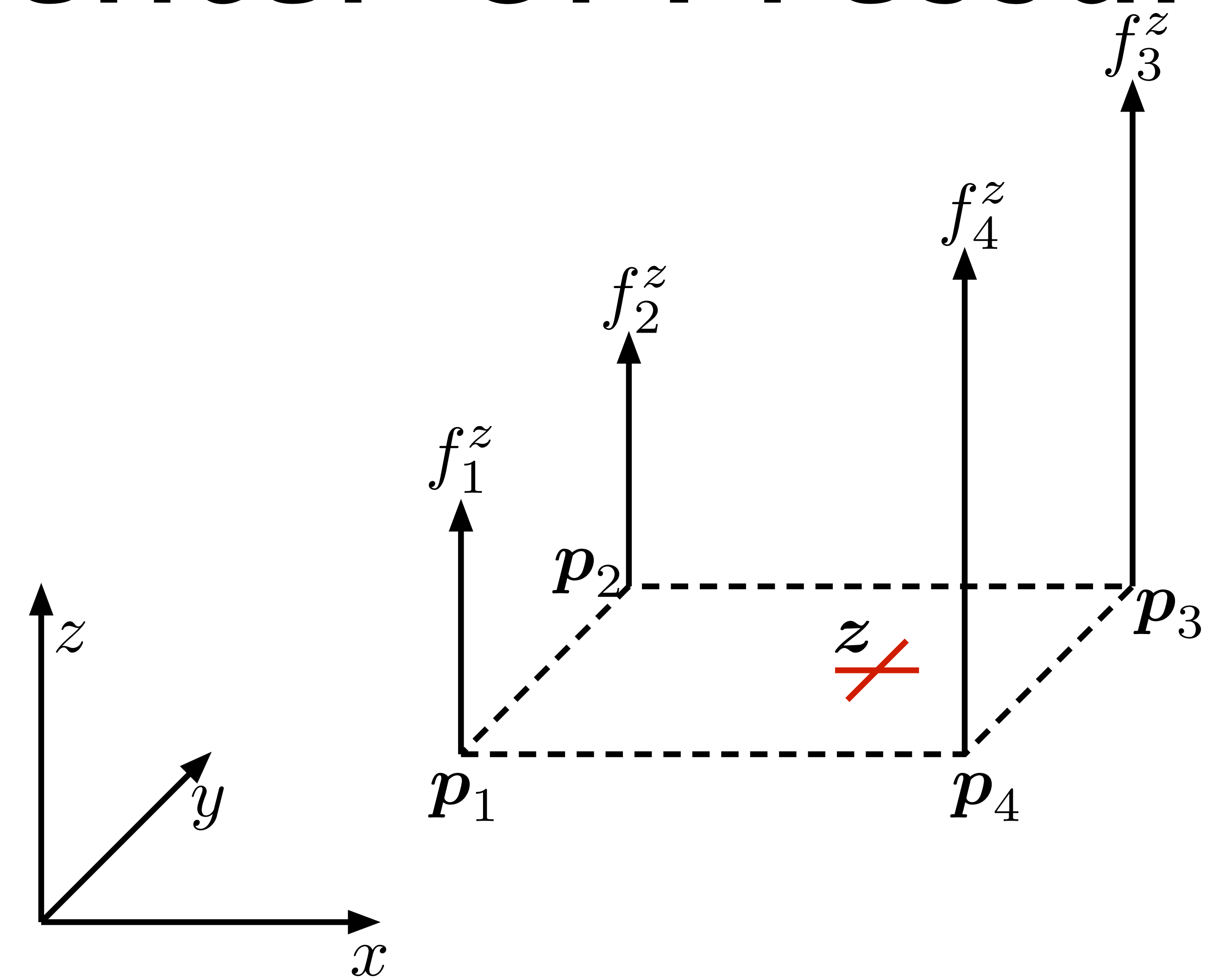
$$L=\sum_k (x_k-c)\times m_k\dot{x}_k+I_k\omega_k$$

On a flat ground, the Center of Pressure

$$\frac{m\, c \times (\ddot{c} + g) + \dot{L}}{m(\ddot{c}^z + g^z)} = \frac{\sum_i p_i \times f_i}{\sum_i f_i^z}$$

$$c^{x,y} - \frac{c^z}{\ddot{c}^z + g^z}(\ddot{c}^{x,y} + g^{x,y}) + \frac{1}{m(\ddot{c}^z + g^z)} S \dot{L}^{x,y} = \frac{\sum_i f_i^z\, p_i^{x,y}}{\sum_i f_i^z}$$

On a flat ground, the Center of Pressure

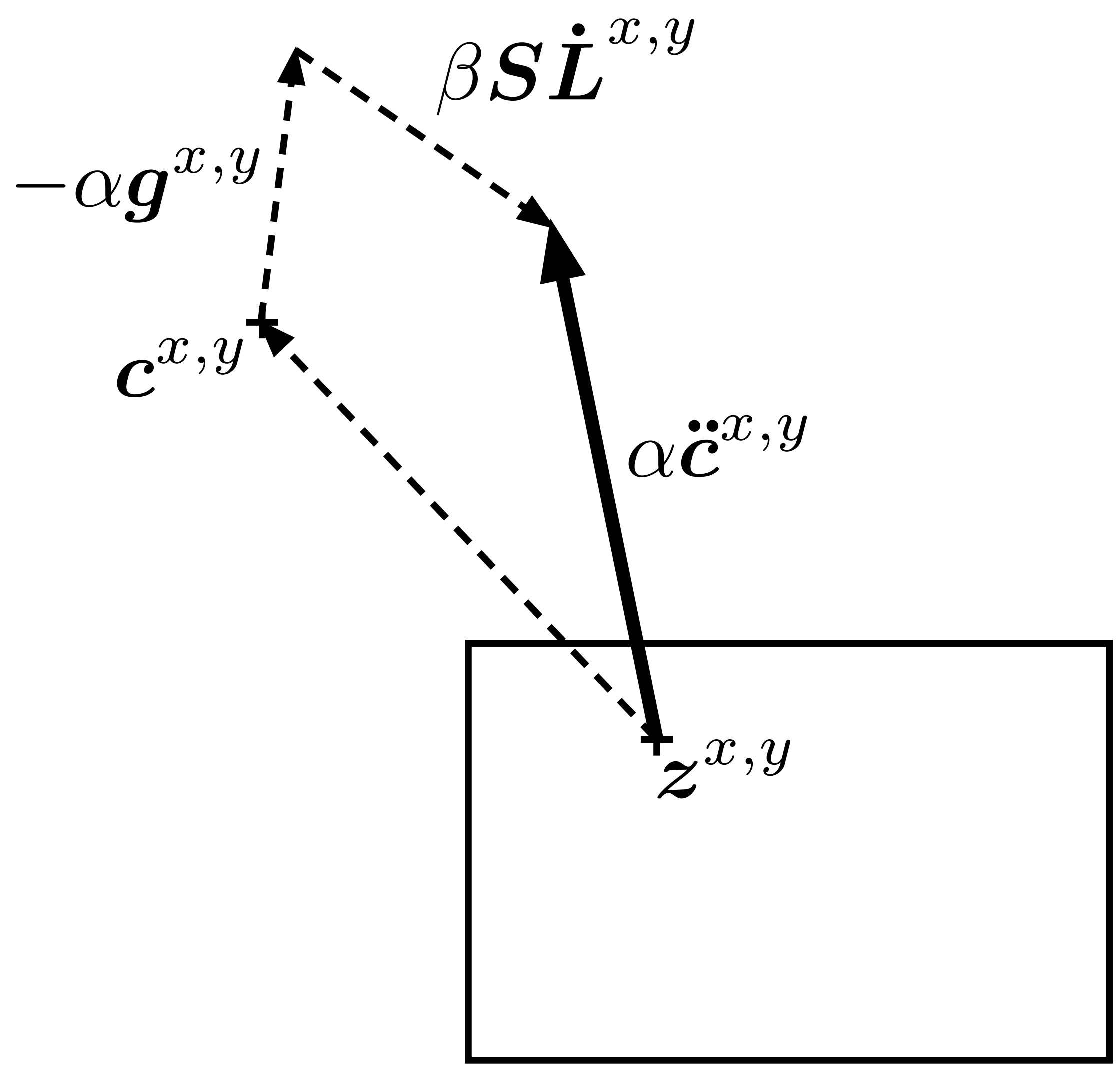


On a flat ground, the Center of Pressure

$$c^{x,y} - \frac{c^z}{\ddot{c}^z + g^z}(\ddot{c}^{x,y} + g^{x,y}) + \frac{1}{m(\ddot{c}^z + g^z)} S\dot{L}^{x,y} = \frac{\sum_i f_i^z \, p_i^{x,y}}{\sum_i f_i^z}$$

$$\frac{c^z}{\ddot{c}^z + g^z}(\ddot{c}^{x,y} + g^{x,y}) = (c^{x,y} - z^{x,y}) + \frac{1}{m(\ddot{c}^z + g^z)} S\dot{L}^{x,y}$$

On a flat ground, the Center of Pressure



On a flat ground, the Center of Pressure

$$c^{x,y} - \frac{c^z}{\ddot{c}^z + g^z}(\ddot{c}^{x,y} + g^{x,y}) + \frac{1}{m(\ddot{c}^z + g^z)} S\dot{L}^{x,y} = \frac{\sum_i f_i^z \, p_i^{x,y}}{\sum_i f_i^z}$$

$$\frac{c^z}{\ddot{c}^z + g^z}(\ddot{c}^{x,y} + g^{x,y}) = (c^{x,y} - z^{x,y}) + \frac{1}{m(\ddot{c}^z + g^z)} S\dot{L}^{x,y}$$

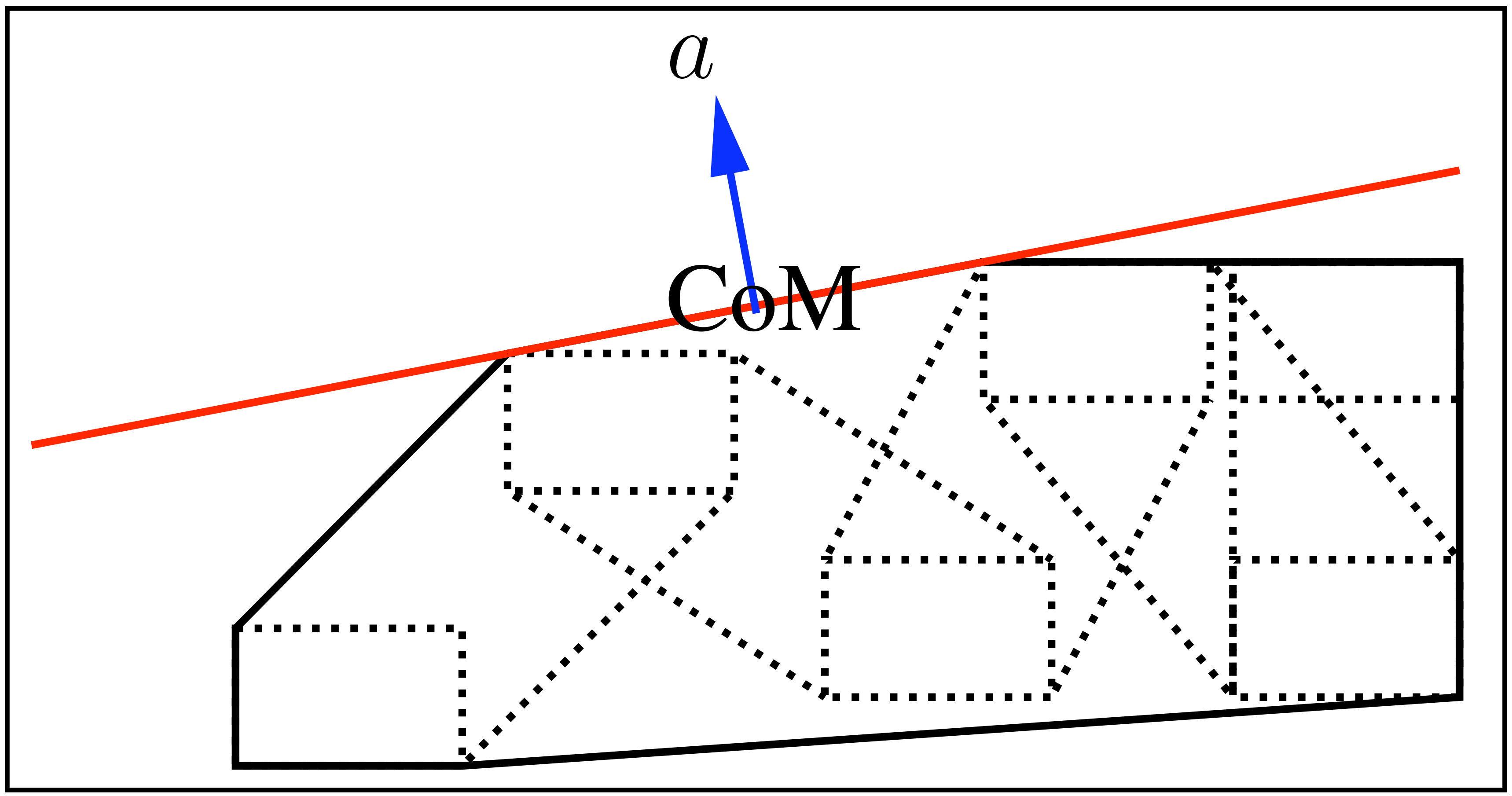
Walking horizontally

$$\frac{c^z}{\ddot{c}^z + g^z}(\ddot{c}^{x,y} + g^{x,y}) = (c^{x,y} - z^{x,y}) + \frac{1}{m(\ddot{c}^z + g^z)} S\dot{L}^{x,y}$$

$$c^{x,y} - \frac{c^z}{g^z}\ddot{c}^{x,y} = z^{x,y}$$

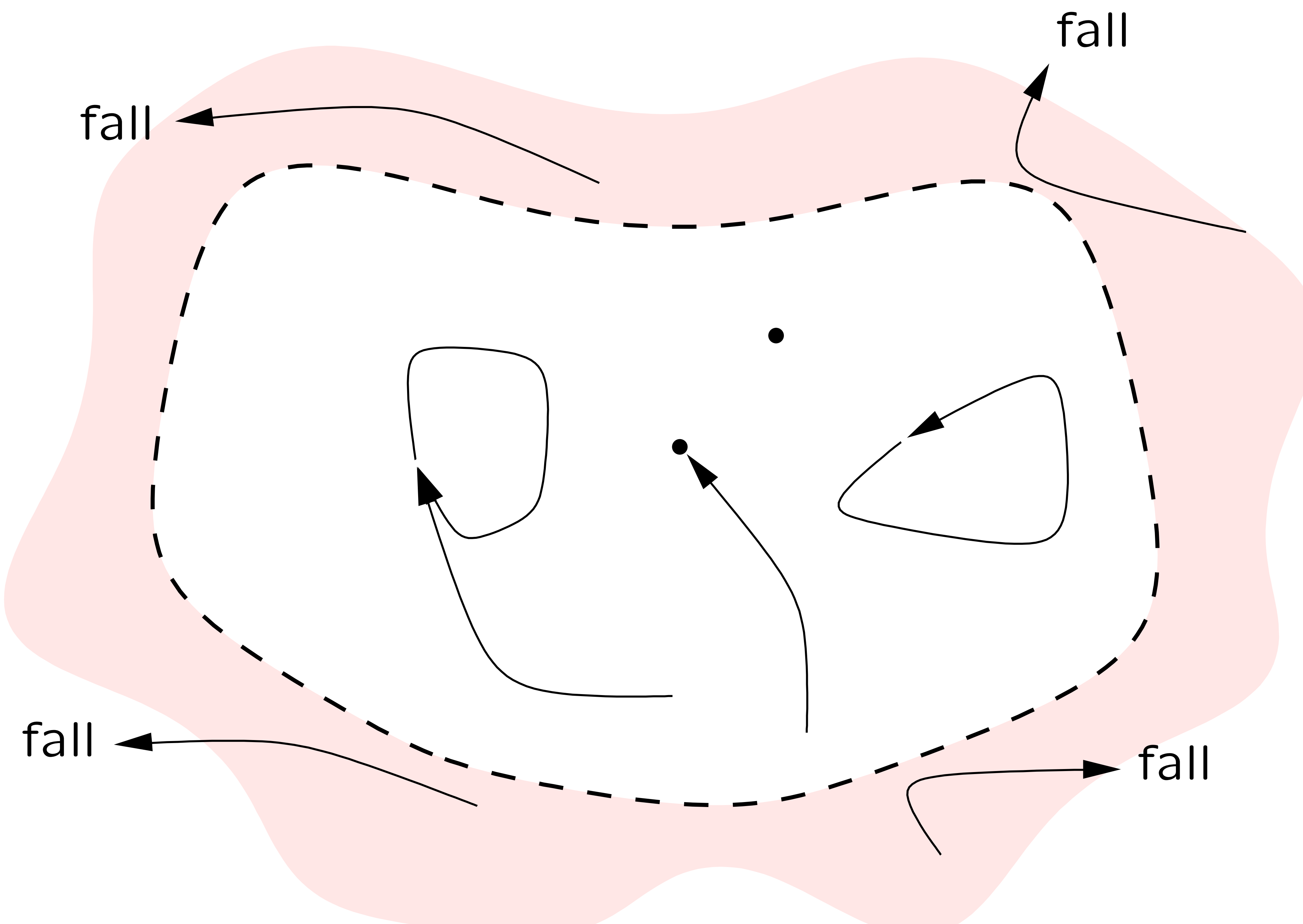
- Not just a «Linear Inverted Pendulum Model»

The dynamics of falling

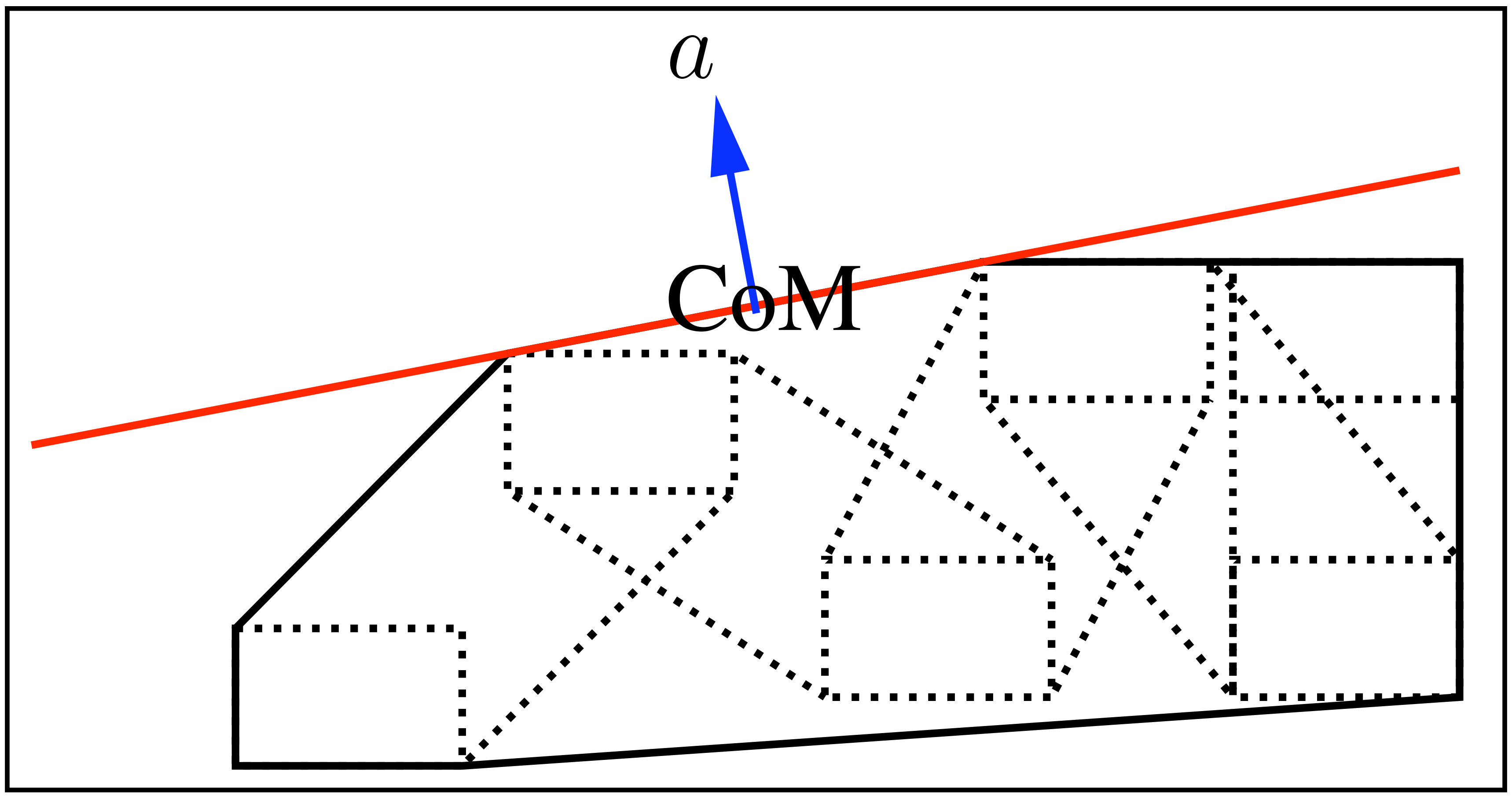


$$a^T(c^{x,y}(t) - c^{x,y}(t_0)) \geq \frac{a^T \dot{c}^{x,y}(t_0)}{\omega} \sinh(\omega(t - t_0))$$

Viability

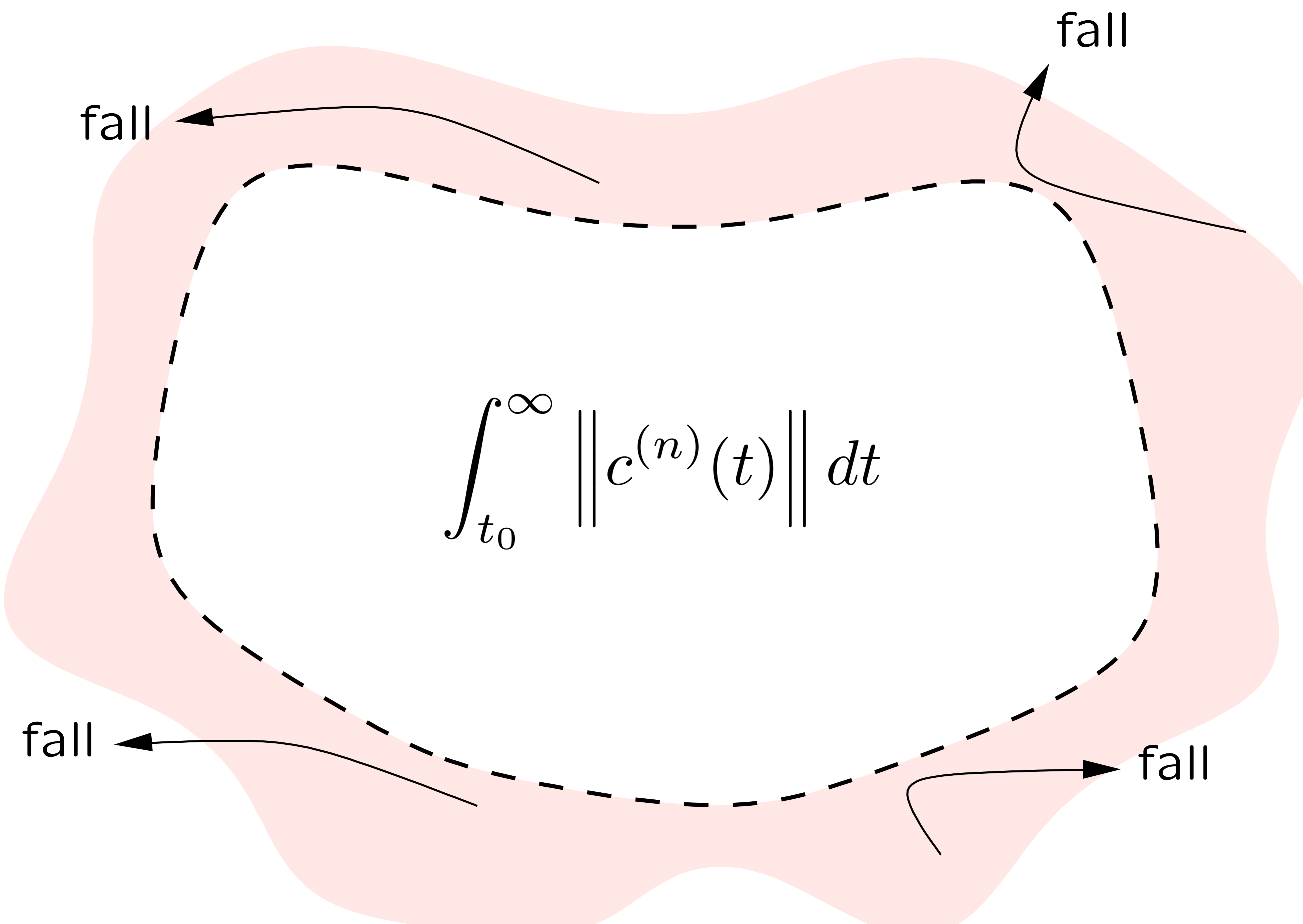


The dynamics of falling



$$a^T(c^{x,y}(t) - c^{x,y}(t_0)) \geq \frac{a^T \dot{c}^{x,y}(t_0)}{\omega} \sinh\left(\omega(t - t_0)\right)$$

Viability, capturability



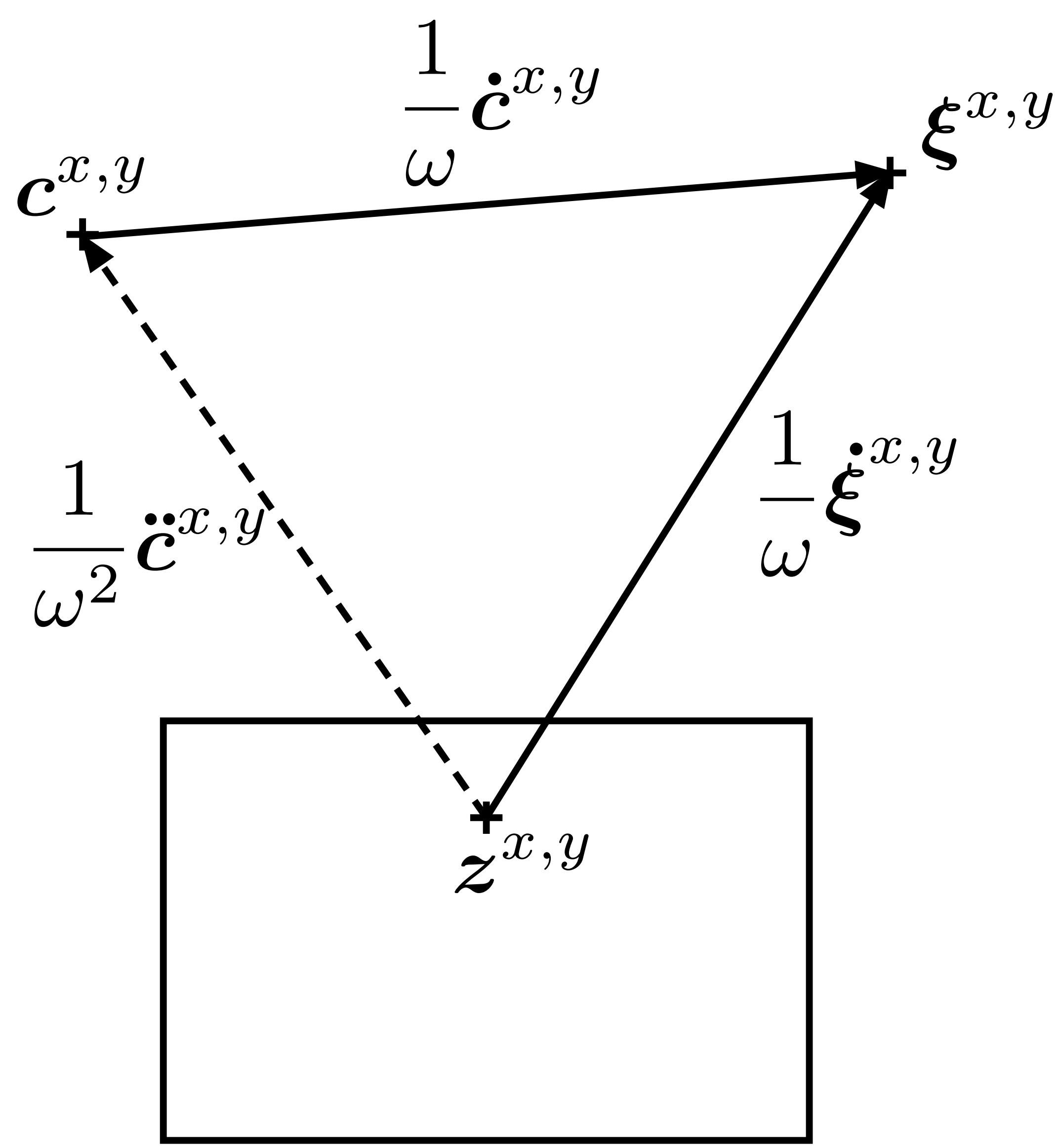
The Capture Point

$$\xi = c + \frac{1}{\omega}\dot{c}$$

$$\dot{c} = \omega(\xi - c)$$

$$\dot{\xi}^{x,y} = \omega(\xi^{x,y} - z^{x,y})$$

The Capture Point



Generation of dynamic walking motions

Early offline schemes

- Trajectory optimization
- Artificial synergy synthesis & ZMP approach

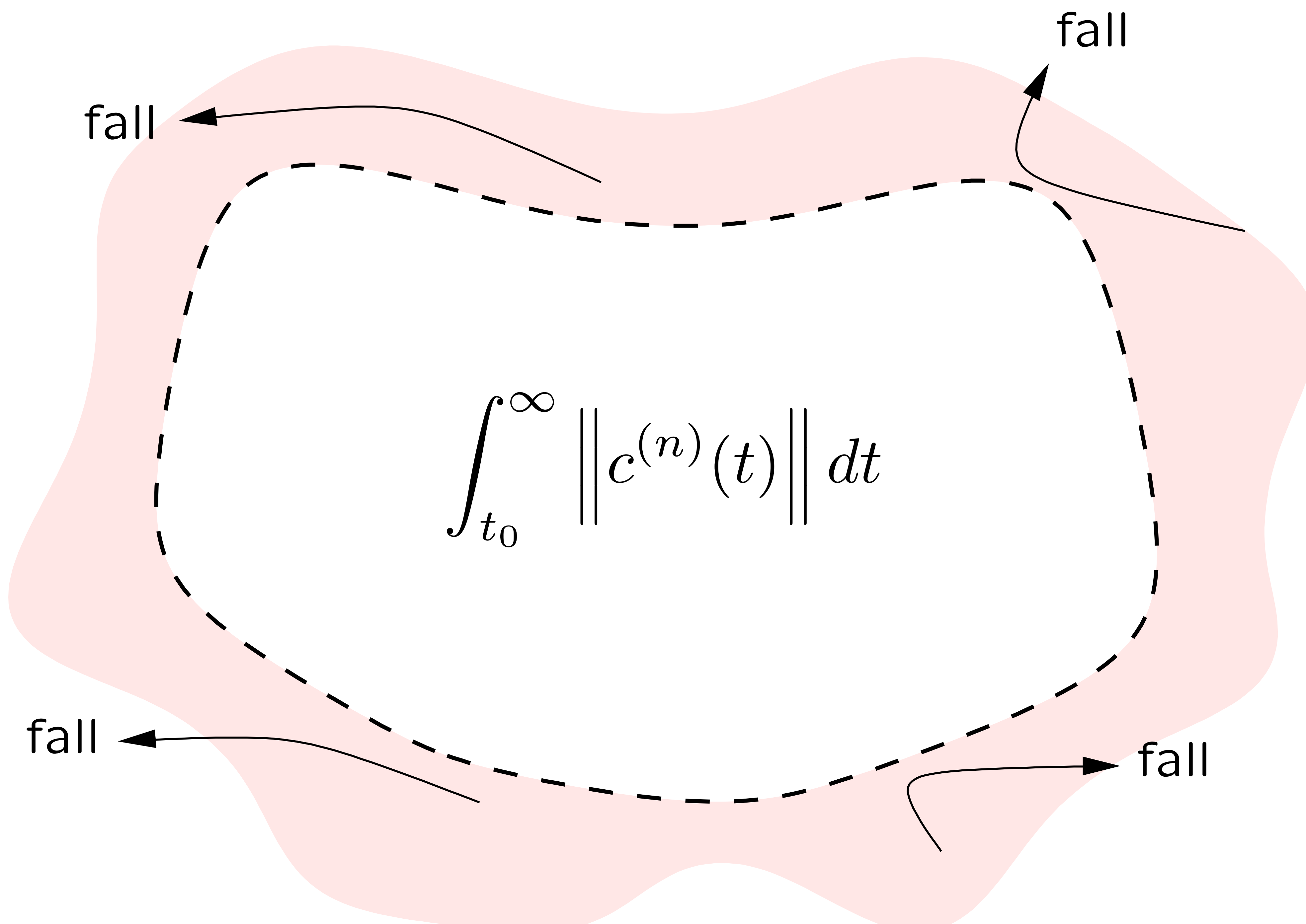
$$\frac{c^z}{\ddot{c}^z + g^z}(\ddot{c}^{x,y} + g^{x,y}) = (c^{x,y} - z^{x,y}) + \frac{1}{m(\ddot{c}^z + g^z)} S\dot{L}^{x,y}$$

- Templates & anchors

Online motion generation

- Necessary for reactivity
- How to make sure you are stable in the long term?

Viability, capturability



Optimal and Model Predictive Control

Optimal feedback

$$x_{k+1} = f(x_k, u_k)$$

$$V^*(x_0) = \min_{u_0,\ldots} \sum_0^\infty l(x_k, u_k)$$

- $u_0^*(x_0)$ is asymptotically stabilizing if the system is controllable
- $V^*(x_0)$ as Lyapunov function

Terminal constraint

- Keerthi 1988 JOTA

$$V_N^*(x_0) = \min_{u_0,\ldots} \sum_{0}^{N-1} l(x_k, u_k) \text{ with } x_N = 0$$

$$V_N^*(x_0) \geq V_{N+1}^*(x_0) \geq \cdots \geq V^*(x_0)$$

$$V_N^*(x_0) \geq l(x_0, u_0^*) + V_N^*(f(x_0, u_0^*))$$

- $V_N^*(x_0)$ as Lyapunov function

Feasibility is sufficient

- Alamir 1999 EJC

$$\text{find } u_0, \dots \text{ such that } x_N = 0$$

- Compute a new plan only in case of a diverging perturbation

Horizon long enough

- Alamir 1995 A

$$V_N^*(x_0) = \min_{u_0,\dots} \sum_0^{N-1} l(x_k, u_k)$$

$$V_N^*(x_0) = l(x_0, u_0^*) + V_N^*(f(x_0, u_0^*)) - l({x'}_N, {u^*}'_N)$$

$$\forall N \geq N_\varepsilon,\ l({x'}_N, {u^*}'_N) < \varepsilon$$

- You can do without <u>explicit</u> terminal cost and constraint with a horizon long enough

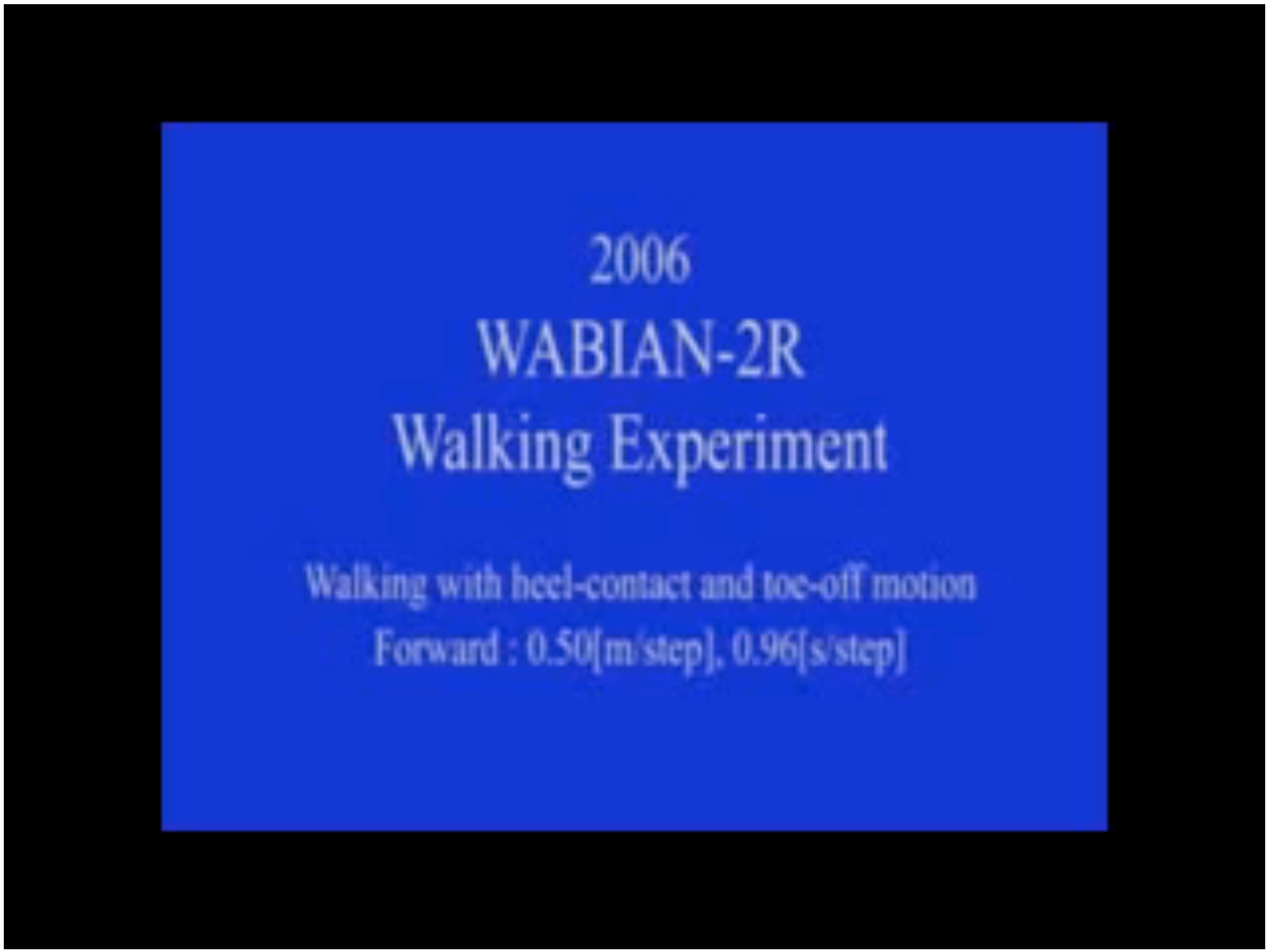
Predefined footsteps, Capturability constraint

Waseda University

$$\frac{\sum m_i(\ddot{c}_i^z + g^z)c_i^{x,y} - m_i\, c_i^z\, \ddot{c}_i^{x,y}}{\sum m_i(\ddot{c}_i^z + g^z)} \longrightarrow z_{ref}^{x,y}$$

- Must always be able to stop within 2 steps.

Waseda University



TUM Johnny/Lola

$$\frac{\sum m_i(\ddot{c}_i^z + g^z)c_i^{x,y} - m_i\, c_i^z\, \ddot{c}_i^{x,y}}{\sum m_i(\ddot{c}_i^z + g^z)} \approx z_{ref}^{x,y}$$

$$c^{x,y}(t + \Delta T) = c_{ref}^{x,y}$$

TUM Johnny/Lola

Autonomous
Humanoid Robot
Lola

at the Hannover Fair 2010

Honda Asimo

$$\frac{\sum m_i(\ddot{c}_i^z + g^z)c_i^{x,y} - m_i\, c_i^z\, \ddot{c}_i^{x,y}}{\sum m_i(\ddot{c}_i^z + g^z)} \approx z_{ref}^{x,y}$$

$$\xi^{x,y}(t + \Delta t) = \xi_{ref}^{x,y}$$

ToDaï H7 & Toyota

$$c^{x,y} - \frac{m\, c^z\, \ddot{c}^{x,y} - \dot{L}^{x,y}}{m(\ddot{c}^z + g^z)} \longrightarrow \overline{p_i^{x,y}}$$

$$c^{x,y}(t + \Delta T) = c^{x,y}_{ref}$$

Sony QRIO

$$\min \sum \left\| c^{x,y} - \frac{c^z}{g^z}\ddot{c}^{x,y} - \overline{p_i^{x,y}} \right\|^2$$

$$c^{x,y}(t+\Delta T) = c^{x,y}_{ref},\ \dot{c}^{x,y}(t+\Delta T) = 0$$

Predefined footsteps,
NO capturability
constraint

Kawada HRP-2

$$\min \sum \| \dddot{c}^{\,x,y} \|^2 + \beta \left\| c^{x,y} - \frac{c^z}{g^z}\ddot{c}^{x,y} + \frac{S\dot{L}^{x,y}}{mg^z} - \overline{p_i^{x,y}} \right\|^2$$

A korean variant

$$\min \sum \|\dddot{c}^{x,y}\|^2 + \beta \left\| c^{x,y} - \frac{c^z}{g^z}\ddot{c}^{x,y} + \frac{S\dot{L}^{x,y}}{mg^z} - \overline{p_i^{x,y}} \right\|^2 + \gamma \|L^{x,y}\|^2$$

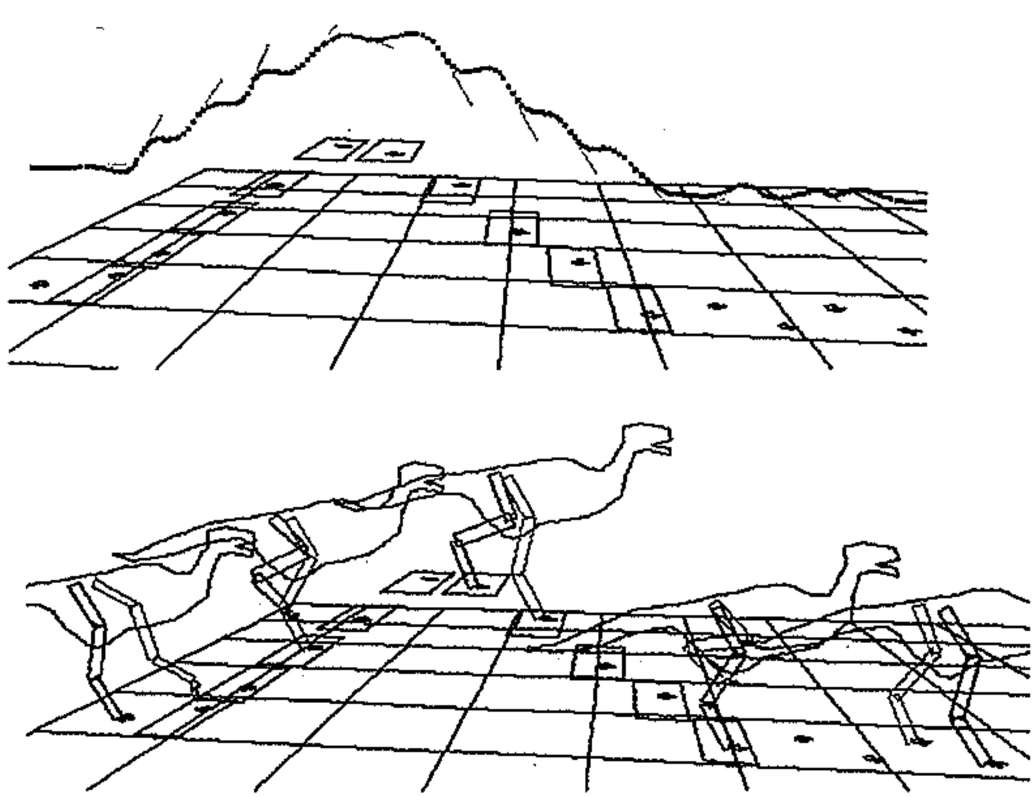
Nao omniwalk

$$\min \sum \| \dddot{c}^{\,x,y} \|^2$$

$$c^{x,y} - \frac{c^z}{g^z} \ddot{c}^{x,y} \in \operatorname{conv} \{ p_i^{x,y} \}$$

EuroGraphics

$$\min \sum \left(\left\| c_k^{x,y,z} - \overline{p_i^{x,y,z}} \right\| - l_{ref} \right)^2 + \left\| c^{x,y} - \frac{c^z \ddot{c}^{x,y}}{\ddot{c}^z + g^z} - \overline{p_i^{x,y}} \right\|^2$$



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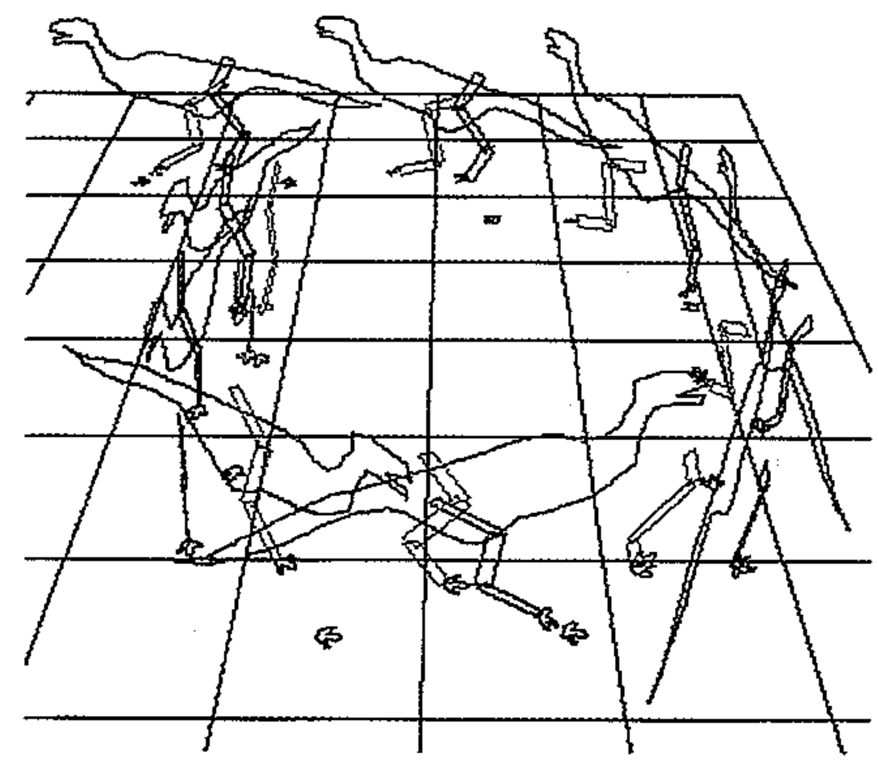


Figure 17: *Dinosaur turning and leaping. (a) footpri*

Adaptive footsteps

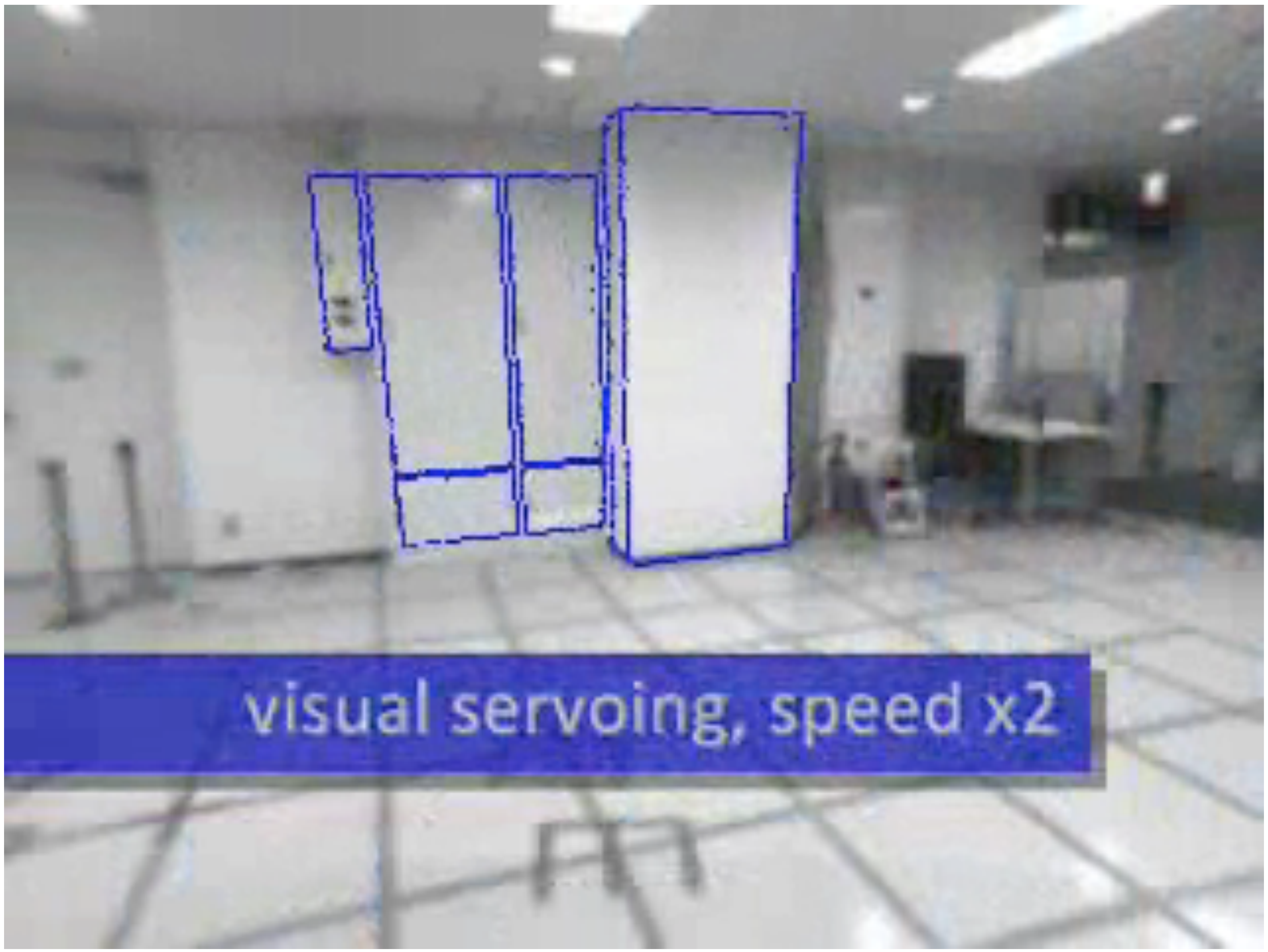
Nao's future algorithm

$$\min \sum \left\| \dot{c}^{x,y} - \dot{c}^{x,y}_{ref} \right\|^2$$

$$c^{x,y} - \frac{c^z}{g^z} \ddot{c}^{x,y} \in \mathrm{conv}\left\{p_i^{x,y}\right\}$$

Walking without thinking about it

Vision feedback



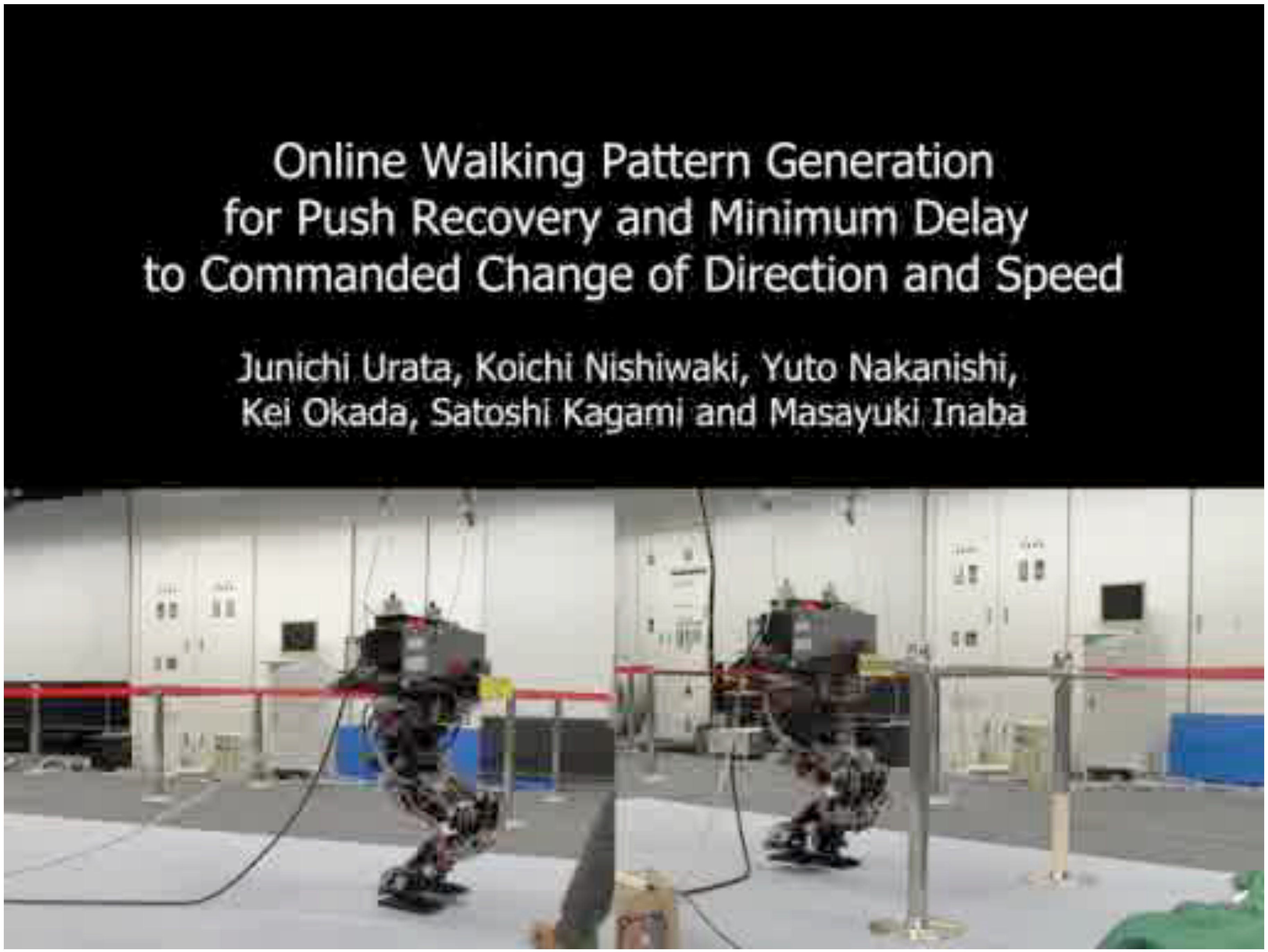
Todaï

$$\xi^{x,y}(t+\Delta t) = \xi^{x,y}_{ref}$$

Todaï


Online Decision of Foot Placement
using Singular LQ Preview Regulation

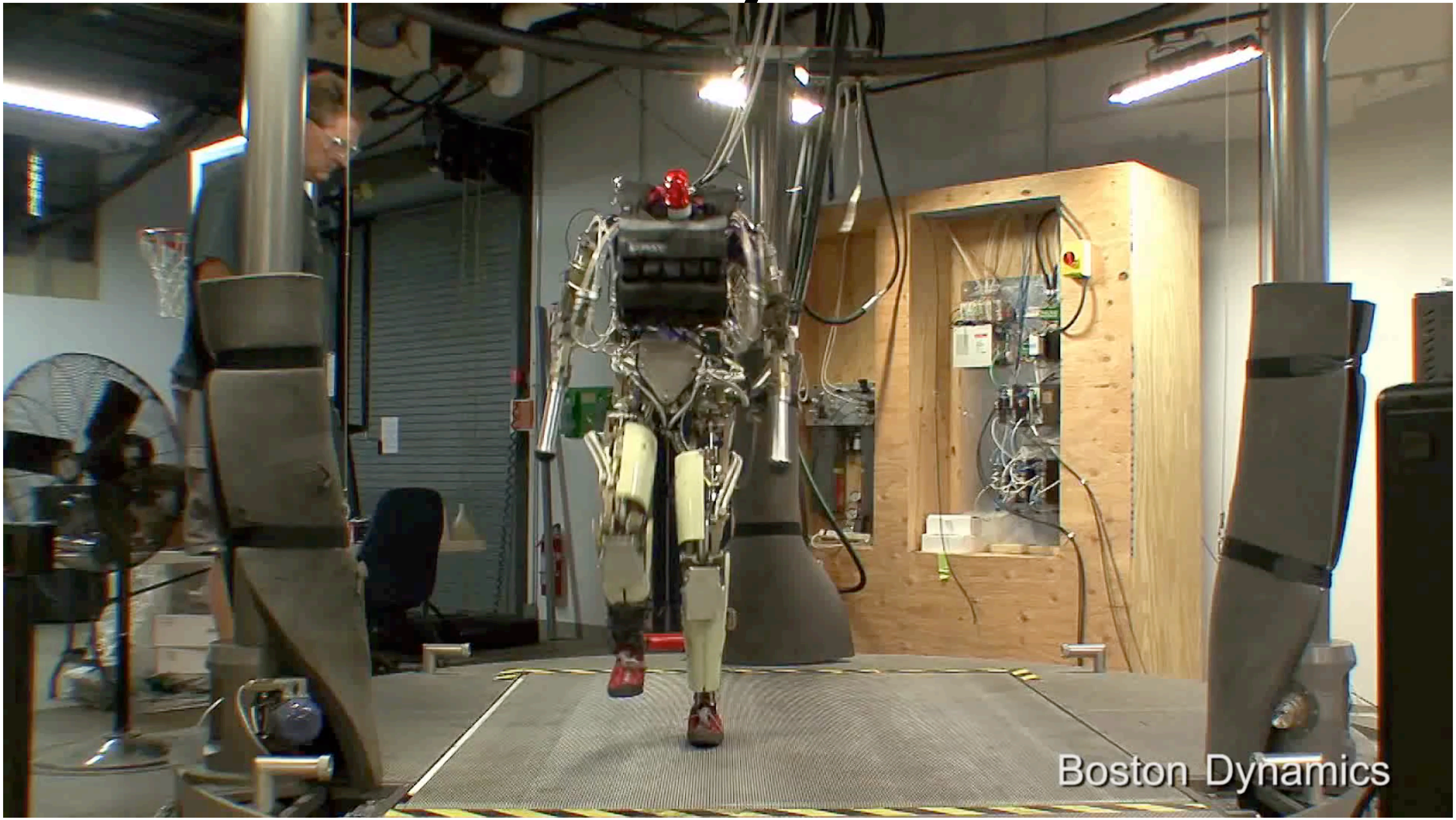
Todaï


Online Walking Pattern Generation
for Push Recovery and Minimum Delay
to Commanded Change of Direction and Speed
Junichi Urata, Koichi Nishiwaki, Yuto Nakanishi,
Kei Okada, Satoshi Kagami and Masayuki Inaba

Key ingredients ?

- Viability & Capturability
- Artificial synergy synthesis
- Model Predictive Control

Boston Dynamics ?



Don’t want/have computing resources ?

Combining simple rules (MIT LegLab)

- Control vertical oscillations
- Control upper body attitude
- Adaptive step placement, «neutral position»

Combining simple rules (biomimetic)

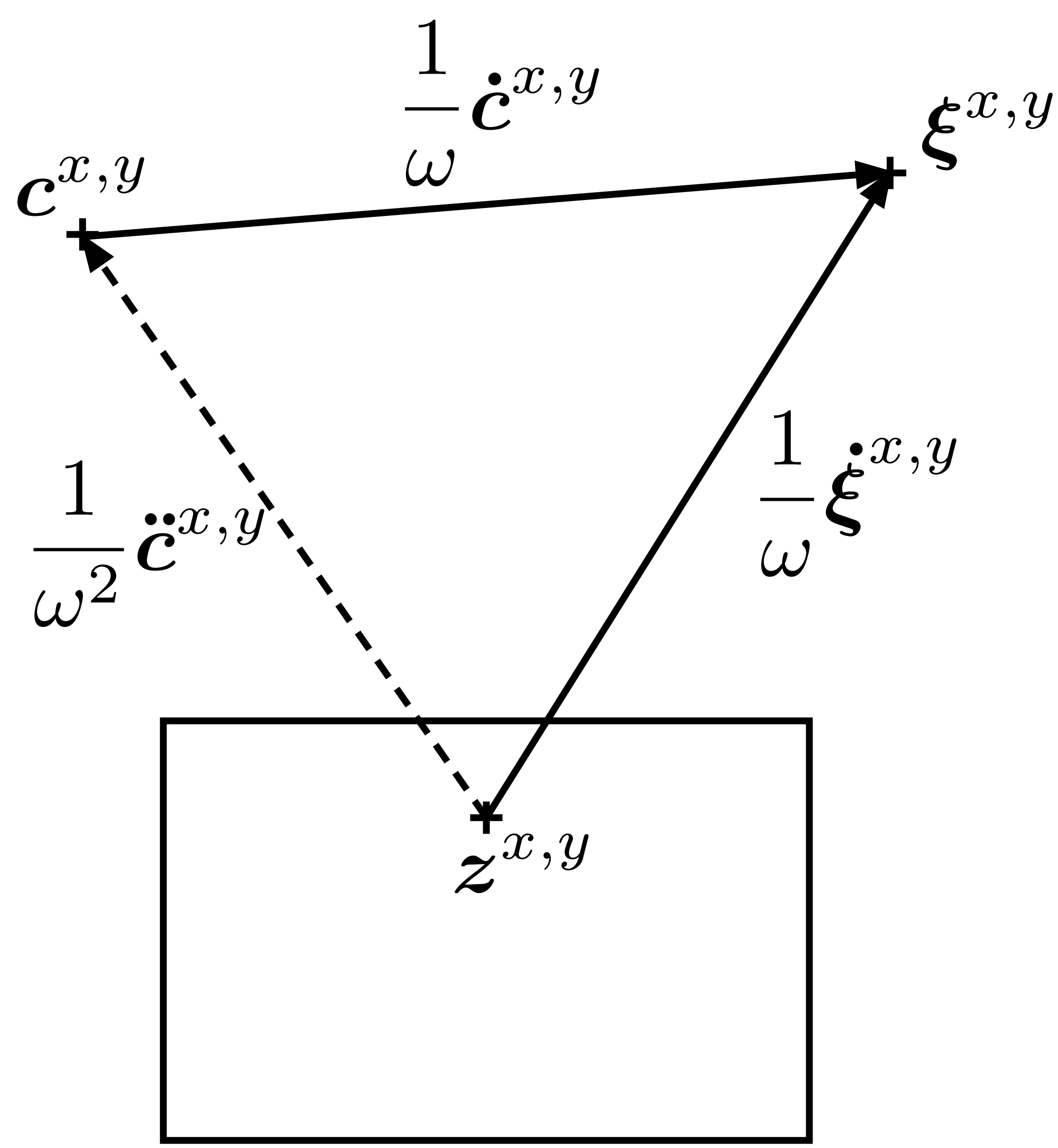
- Central Pattern Generators (oscillators)
- Control upper body attitude
- <u>Adaptive step placement</u>

Motion and Force Control

Whole body motion

- Inverse Kinematics + joint control
- Virtual Model Control
- Task Function Approach
- Operational Space Control

CoM motion control



CoM motion control

$$\dot{c} = \omega(\xi - c)$$

$$\dot{\xi}^{x,y} = \omega(\xi^{x,y} - z^{x,y})$$

$$z^{x,y} = c_{ref}^{x,y} + k(\xi^{x,y} - c_{ref}^{x,y})$$

$$\dot{\xi}^{x,y} = \omega(k-1)(c_{ref}^{x,y} - \xi^{x,y})$$

Contact force control

- Damping oscillations

$$\dot{z} = \omega_z(z_d - z)$$

$$z_d^{x,y} = c_{ref}^{x,y} + k(\xi^{x,y} - c_{ref}^{x,y}) + k'(z^{x,y} - c_{ref}^{x,y})$$

Contact force control

Numerical implementation

People who are really serious about software should make their own hardware.

Alan Kay
(invented Object Oriented Programming at Xerox PARC)

People who are really serious about control algorithms should make their own numerical solver.

Pierre-Brice Wieber
(invented not much yet at INRIA Grenoble)

Discretized trajectories

- Nishiwaki 2002 IROS: 52–104 samples over 3 steps, 2.6–5.2 s / 50 ms
- Buschmann 2007 ICHR: 20–30 pieces of cubic spline over 3 steps ≈ 100 ms period
- Morisawa 2006 ICHR: 7 pieces of quartic or quintic + exponential over 2 steps ≈ 300 ms

Discretized trajectories

- Takenaka 2009 IROS: 14 pieces of line + exponential over 2 steps ≈ 70–130 ms
- Pratt 2006 ICHR: 2 pieces of exponentials over 1 step

Discretized trajectories

- Van de Panne 1997 EG: 2 pieces of cubic spline over 2 steps
- Kajita 2003 ICRA: 320–640 pieces with constant jerk over 2–4 steps, 1.6–3.2 s / 5 ms
- Herdt 2010 RSJAR: 16 pieces with constant jerk over 2 steps, 1.6 s / 100 ms

Discretized trajectories

- Like Kajita and Buschmann, let's use cubic splines (piecewise constant jerk) for the CoM
- We can bound the overshoot of the CoP:

$$z^{x,y} - z^{x,y}_{\max} \leq \frac{1}{8}\dddot{c}^{x,y}_{\max}\delta t^2 \approx \frac{g^z \|z^{x,y} - c^{x,y}\|_{\max}}{8c^z}\delta t^2 \approx \frac{1}{2}\delta t^2$$

- 100 ms appears fair enough in this linear

Detailed formulas

Herdt 2010 RSJAR

$$\dddot{C}_k = \begin{bmatrix} \dddot{c}_k \\ \vdots \\ \dddot{c}_{k+N-1} \end{bmatrix}$$

$$\dot{C}_{k+1} = \begin{bmatrix} \dot{c}_{k+1} \\ \vdots \\ \dot{c}_{k+N} \end{bmatrix} = S_v \begin{bmatrix} c_k \\ \dot{c}_k \\ \ddot{c}_k \end{bmatrix} + U_v \dddot{C}_k$$

$$Z^x_{k+1} = \begin{bmatrix} z^x_{k+1} \\ \vdots \\ z^x_{k+N} \end{bmatrix} = S_z \begin{bmatrix} c^x_k \\ \dot{c}^x_k \\ \ddot{c}^x_k \end{bmatrix} + U_z \dddot{C}^x_k$$

Detailed formulas

$$S_v = \begin{bmatrix} 0 & 1 & T \\ \vdots & \vdots & \vdots \\ 0 & 1 & NT \end{bmatrix} \quad U_v = \begin{bmatrix} T^2/2 & 0 & 0 \\ \vdots & \ddots & 0 \\ (1{+}2N)\,T^2/2 & \dots & T^2/2 \end{bmatrix}$$

$$S_z = \begin{bmatrix} 1 & T & T^2/2 - h/g \\ \vdots & \vdots & \vdots \\ 1 & NT & N^2T^2/2 - h/g \end{bmatrix} \quad U_z = \begin{bmatrix} T^3/6 - Th/g & 0 & 0 \\ \vdots & \ddots & 0 \\ (1{+}3N{+}3N^2)\,T^3/6 - Th/g & \dots & T^3/6 - Th/g \end{bmatrix}$$

Cost function

$$\min \;\; \frac{\alpha}{2}\left\|\dddot{C}_{k}^{x,y}\right\|^2 + \frac{\beta}{2}\left\|\dot{C}_{k+1}^{x,y} - \dot{C}_{ref}^{x,y}\right\|^2 + \frac{\gamma}{2}\left\|Z_{k+1}^{x,y} - F_{k+1}^{x,y}\right\|^2$$

$$F_{k+1} = V_{k+1} f_k + \bar{V}_{k+1}\bar{F}_{k+1}$$

$$V_{k+1} = \begin{bmatrix} 1 \\ \vdots \\ 1 \\ 0 \\ \vdots \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \qquad \bar{V}_{k+1} = \begin{bmatrix} 0 & 0 & \\ \vdots & \vdots & \\ 0 & 0 & \\ 1 & 0 & \\ \vdots & \vdots & \\ 1 & 0 & \\ 0 & 1 & \\ \vdots & \vdots & \\ 0 & 1 & \ddots \end{bmatrix}$$

Cost function

$$\min \;\; \frac{\alpha}{2}\left\|\dddot{C}_k^{x,y}\right\|^2 + \frac{\beta}{2}\left\|\dot{C}_{k+1}^{x,y} - \dot{C}_{ref}^{x,y}\right\|^2 + \frac{\gamma}{2}\left\|Z_{k+1}^{x,y} - F_{k+1}^{x,y}\right\|^2$$

$$x_k = \begin{bmatrix} \dddot{C}_k^x \\ \bar{F}_{k+1}^x \\ \dddot{C}_k^y \\ \bar{F}_{k+1}^y \end{bmatrix} \qquad \min_{x_k} \;\; \frac{1}{2} x_k^T Q_k x_k + p_k^T x_k \qquad Q_k = \begin{bmatrix} Q'_k & 0 \\ 0 & Q'_k \end{bmatrix}$$

$$Q'_k = \begin{bmatrix} \alpha I + \beta U_v^T U_v + \gamma U_z^T U_z & -\gamma U_z^T \bar{V}_{k+1} \\ -\gamma \bar{V}_{k+1}^T U_z & \gamma \bar{V}_{k+1}^T \bar{V}_{k+1} \end{bmatrix}$$

Constraints on the CoP

$$l \leq R(F_{k+1}^{\theta}) \begin{bmatrix} Z_{k+1}^{x} - F_{k+1}^{x} \\ Z_{k+1}^{y} - F_{k+1}^{y} \end{bmatrix} \leq u$$

$$l \leq R(F_{k+1}^{\theta}) \left(\begin{bmatrix} U_z & -\bar{V}_{k+1} & 0 & 0 \\ 0 & 0 & U_z & -\bar{V}_{k+1} \end{bmatrix} x_k + \begin{bmatrix} S_z \hat{c}_k^{x} - V_{k+1} f_k^{x} \\ S_z \hat{c}_k^{y} - V_{k+1} f_k^{y} \end{bmatrix} \right) \leq u$$

$$x_k = \begin{bmatrix} \dddot{C}_k^{x} \\ \bar{F}_{k+1}^{x} \\ \dddot{C}_k^{y} \\ \bar{F}_{k+1}^{y} \end{bmatrix}$$

Trivial constraints

$$l \leq R(F^{\theta}_{k+1}) \begin{bmatrix} Z^x_{k+1} - F^x_{k+1} \\ Z^y_{k+1} - F^y_{k+1} \end{bmatrix} \leq u$$

$$\begin{pmatrix} Z'^x_{k+1} \\ Z'^y_{k+1} \end{pmatrix} = R(F^{\theta}_{k+1}) \begin{bmatrix} Z^x_{k+1} - F^x_{k+1} \\ Z^y_{k+1} - F^y_{k+1} \end{bmatrix}$$

$$x'_k = \begin{bmatrix} Z'^x_k \\ \bar{F}^x_{k+1} \\ Z'^y_k \\ \bar{F}^y_{k+1} \end{bmatrix}$$

How to solve a Quadratic Program

Unconstrained

$$\min_x \ \frac{1}{2}x^T Q x + p^T x$$

$$Qx + p = 0$$

Equality constrained

$$\min_x \; \frac{1}{2}x^T Q x + p^T x$$
$$\text{s.t. } Ax + b = 0$$

$$Qx + p = A^T \lambda$$

$$\begin{pmatrix} Q & A^T \\ A & 0 \end{pmatrix} \begin{pmatrix} x \\ -\lambda \end{pmatrix} + \begin{pmatrix} p \\ b \end{pmatrix} = 0$$

Trivially constrained

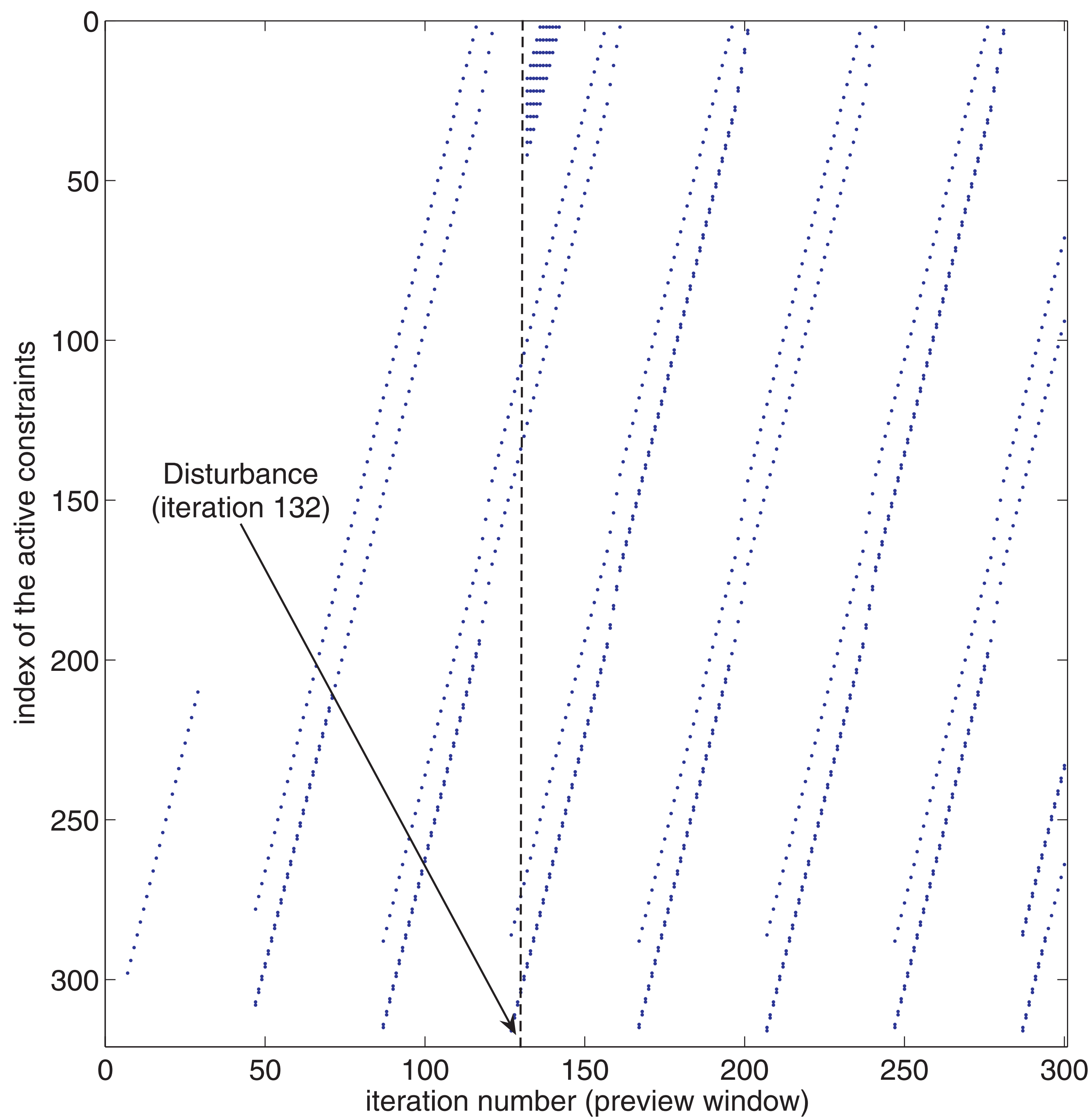
$$\min_x \ \frac{1}{2}x^TQx + p^Tx$$

$$\text{s.t. } \underline{x} + b = 0 \qquad\qquad (\underline{x} = Ex)$$

$$Qx + p = E^T\underline{\lambda}$$

$$\bar{Q}\bar{x} + \bar{p} = 0$$

Active constraints



Computation time