Deutsches Zentrum für Luft- und Raumfahrt
German Aerospace Center

Feedback control of humanoid robots: balancing and walking

Dr.-Ing. Christian Ott

German Aerospace Center (DLR)

Institute for Robotics and Mechatronics

HELMHOLTZ ASSOCIATION

Overview

Part 0: Short overview of (biped robots at) DLR

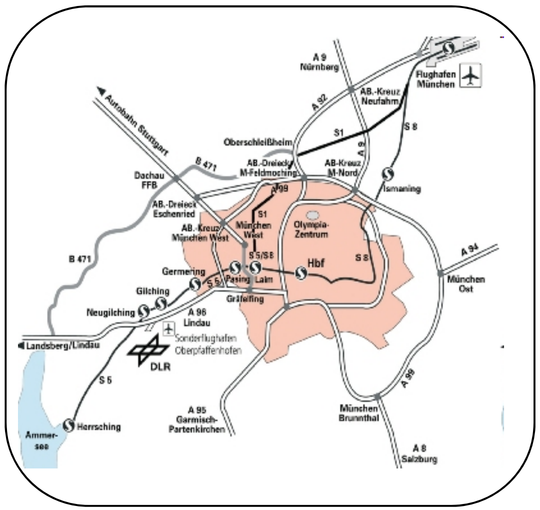
Part I: Modeling

Part II: Balancing

Part III: Walking Control

German Aerospace Center (DLR)

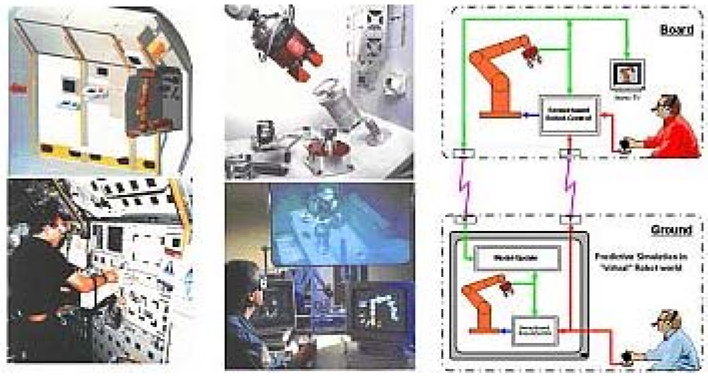
- National research laboratory
- Part of the Helmholtz association
- Research fields:
  - aerospace,
  - space technologies,
  - energy and traffic
- ~6300 researchers
- 13 locations
- 29 institutes



Institute of Robotics and Mechatronics: Former director: Prof. Hirzinger
Director: Prof. Albu-Schäffer

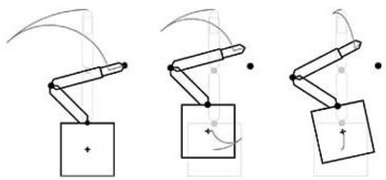
Institute of Robotics & Mechatronics

Space Robotics

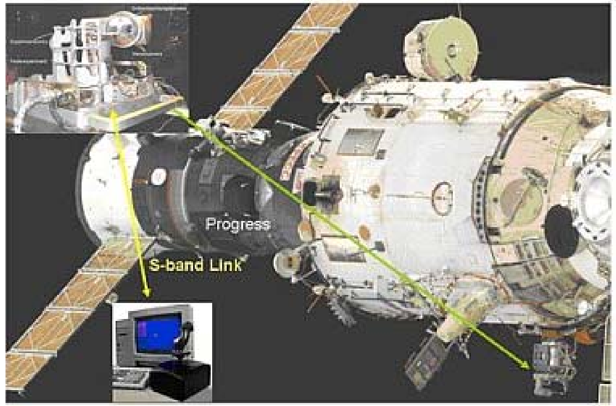


ROTEX 1993

ESS



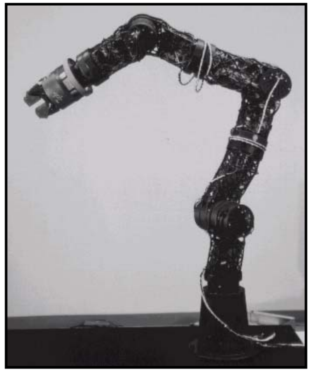
GETEX



ROKVISS 2007-2010

Manipulation

LBR-I,199x



Torque sensors at the power output after the bearings

LBR-II 1999/2000

Torque sensors after the gears, but before the last bearings

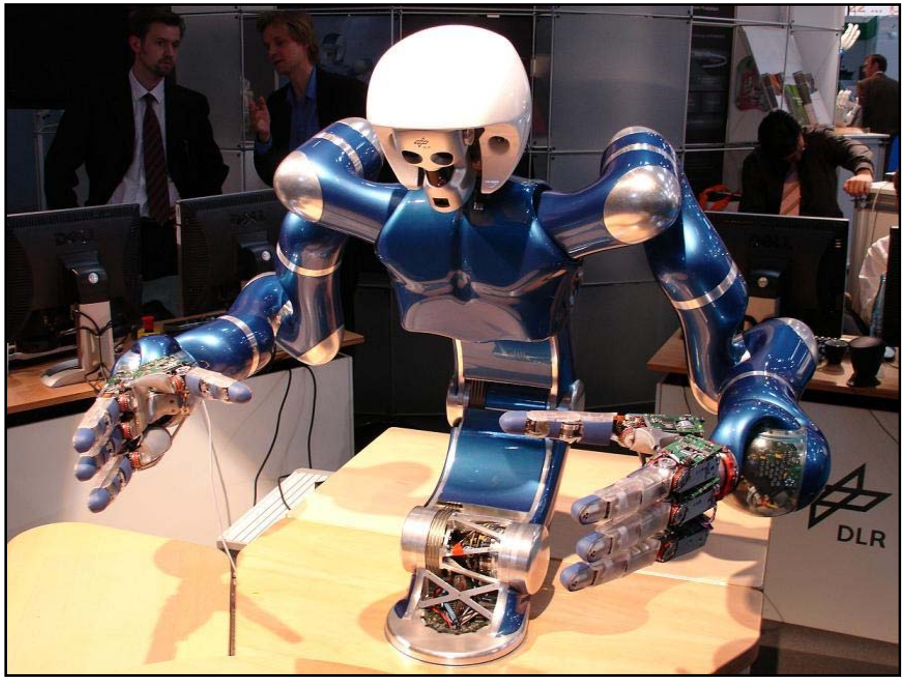
LBR-M, 2003

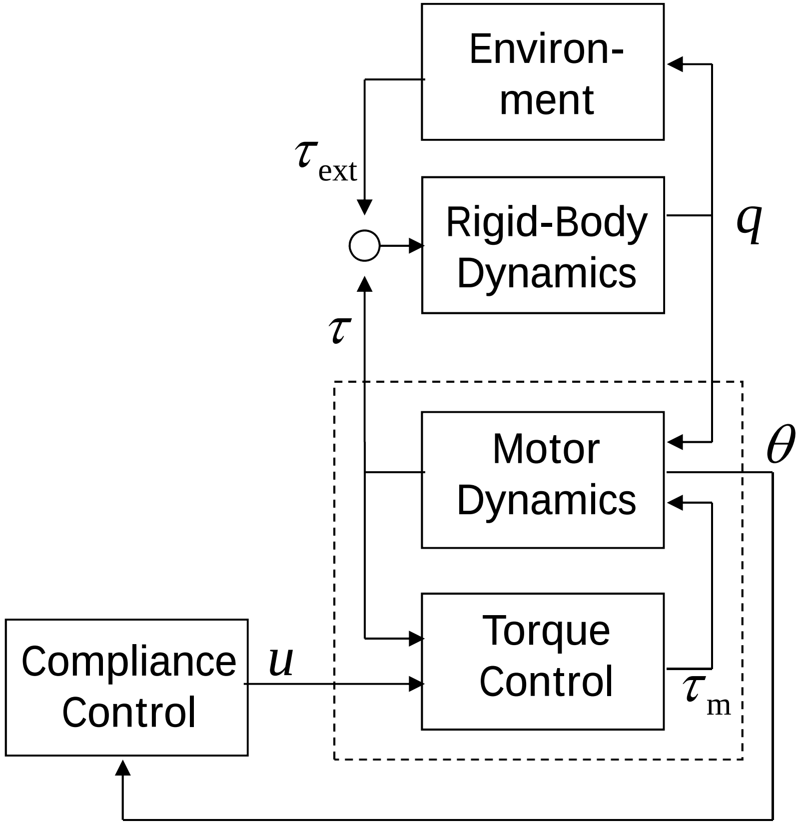
Modular design, load/weight ratio ~ 1:1

Commercial torque controlled arm (KUKA)

Compliant Manipulation

Joint torque sensing & control for manipulation

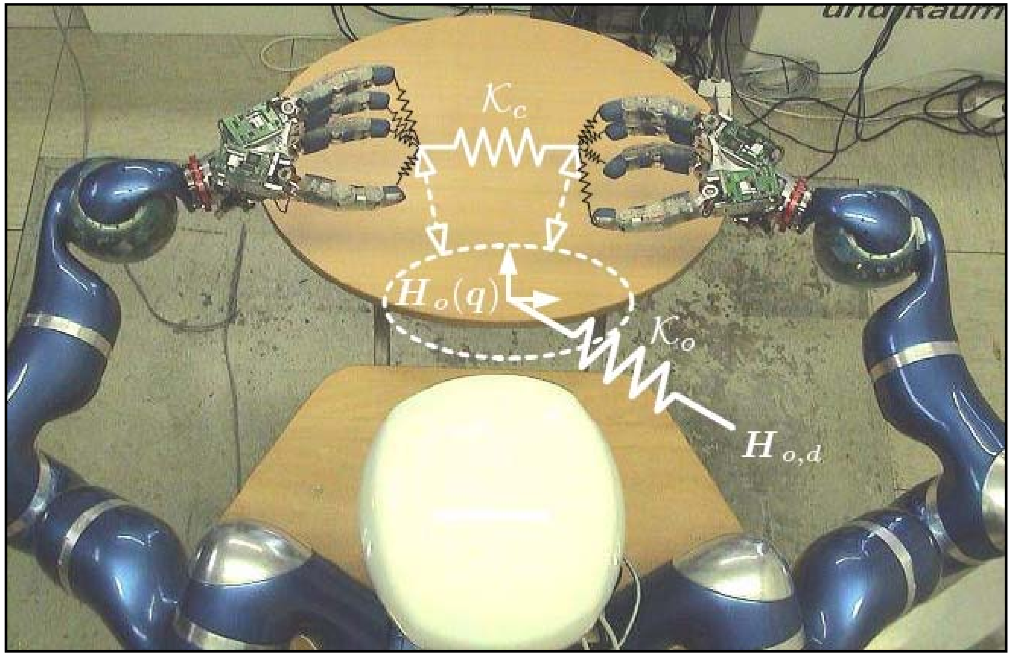


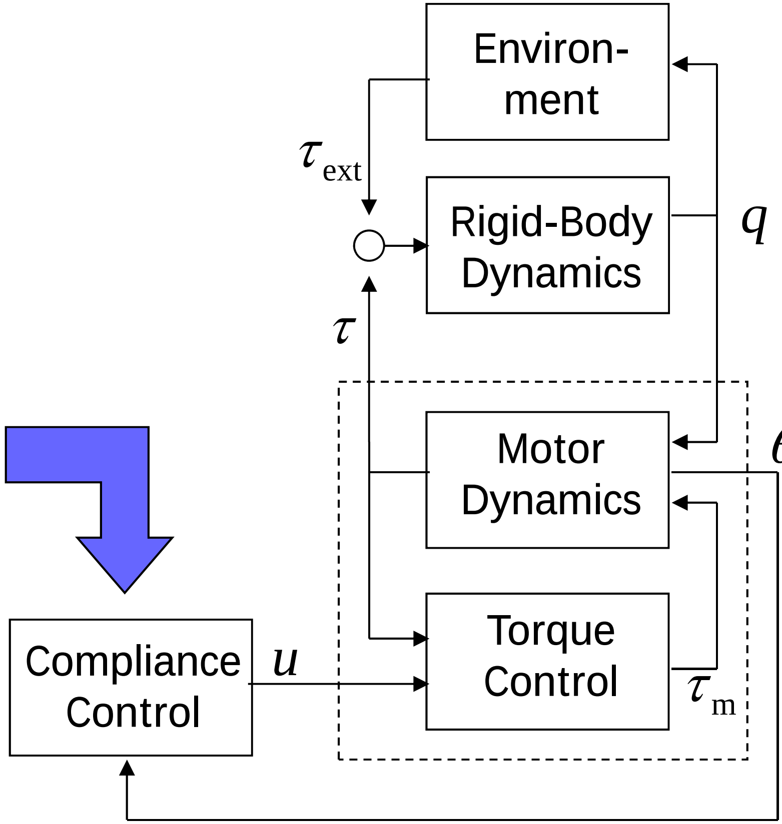


Robustness:
Passivity Based Control

Performance:
Joint Torque Feedback
(noncollocated)

Compliant Manipulation





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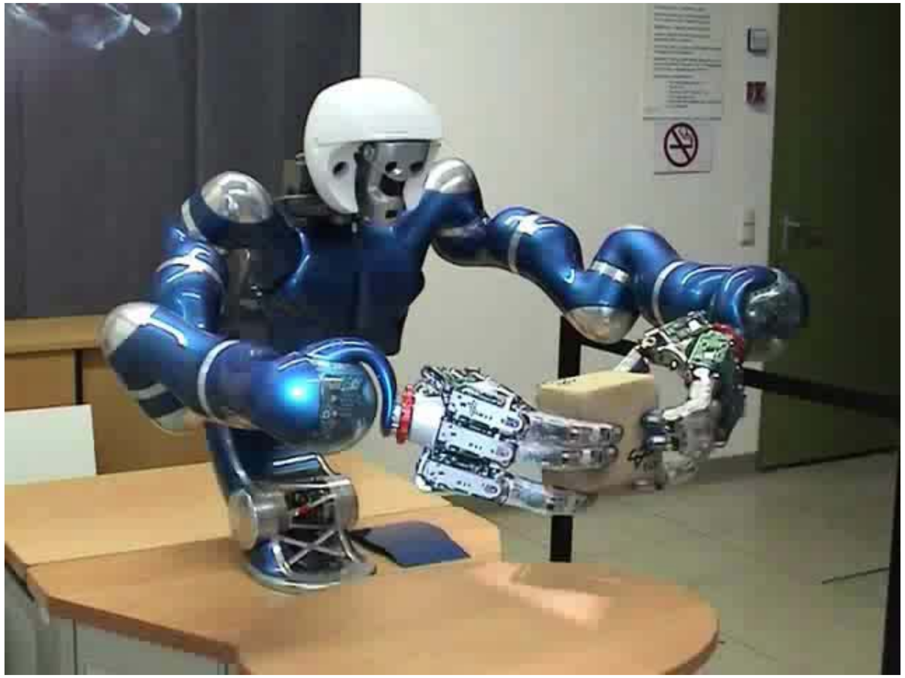

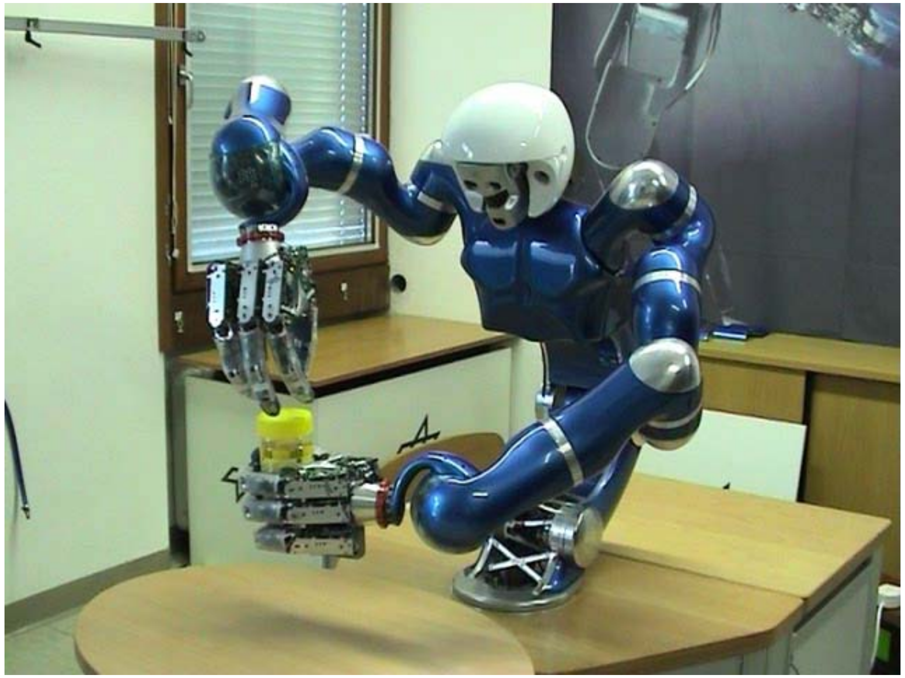

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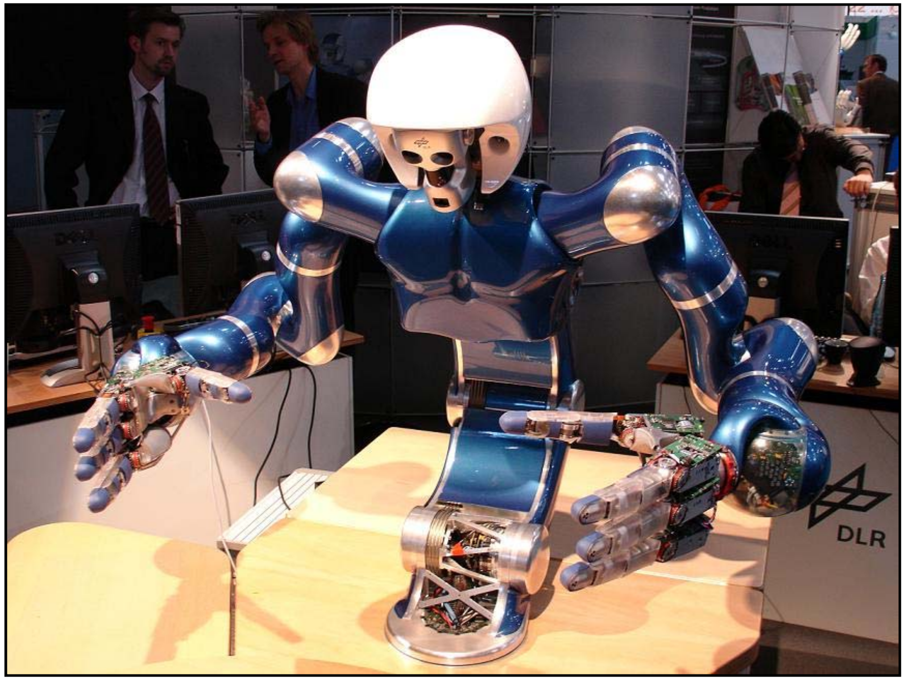
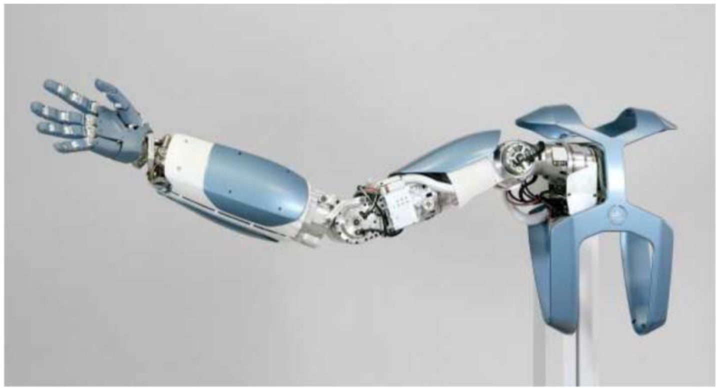
2009-2013

DLR

Joint torque sensing & control for manipulation



Biped Robot



Anthropomorphic Hand-Arm System
[Grebenstein, Albu-Schäffer et al, Humanoids 2010]

- Compliant actuation
- Antagonistic actuation for fingers
- Variable stiffness actuation in arm
- Robustness to shocks and impacts

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Bipedal Walking Robots at DLR

DLR

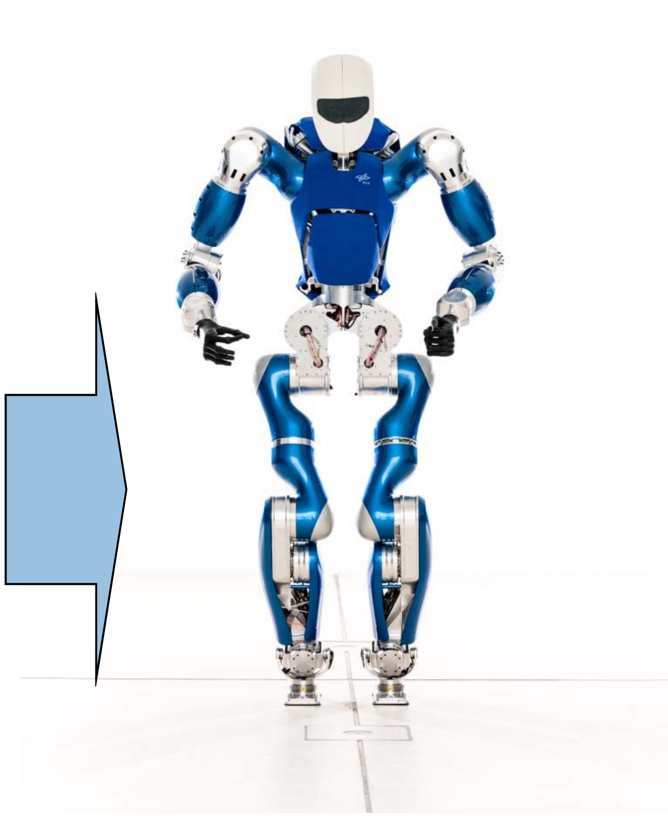
- Drive technology of the DLR arm
  → Allow for position controlled walking (ZMP) and joint torque control!

- Small foot design: 19 x 9,5 cm

- Sensors:
  - joint torque sensors
  - force/torque sensors in the feet
  - IMU in the trunk

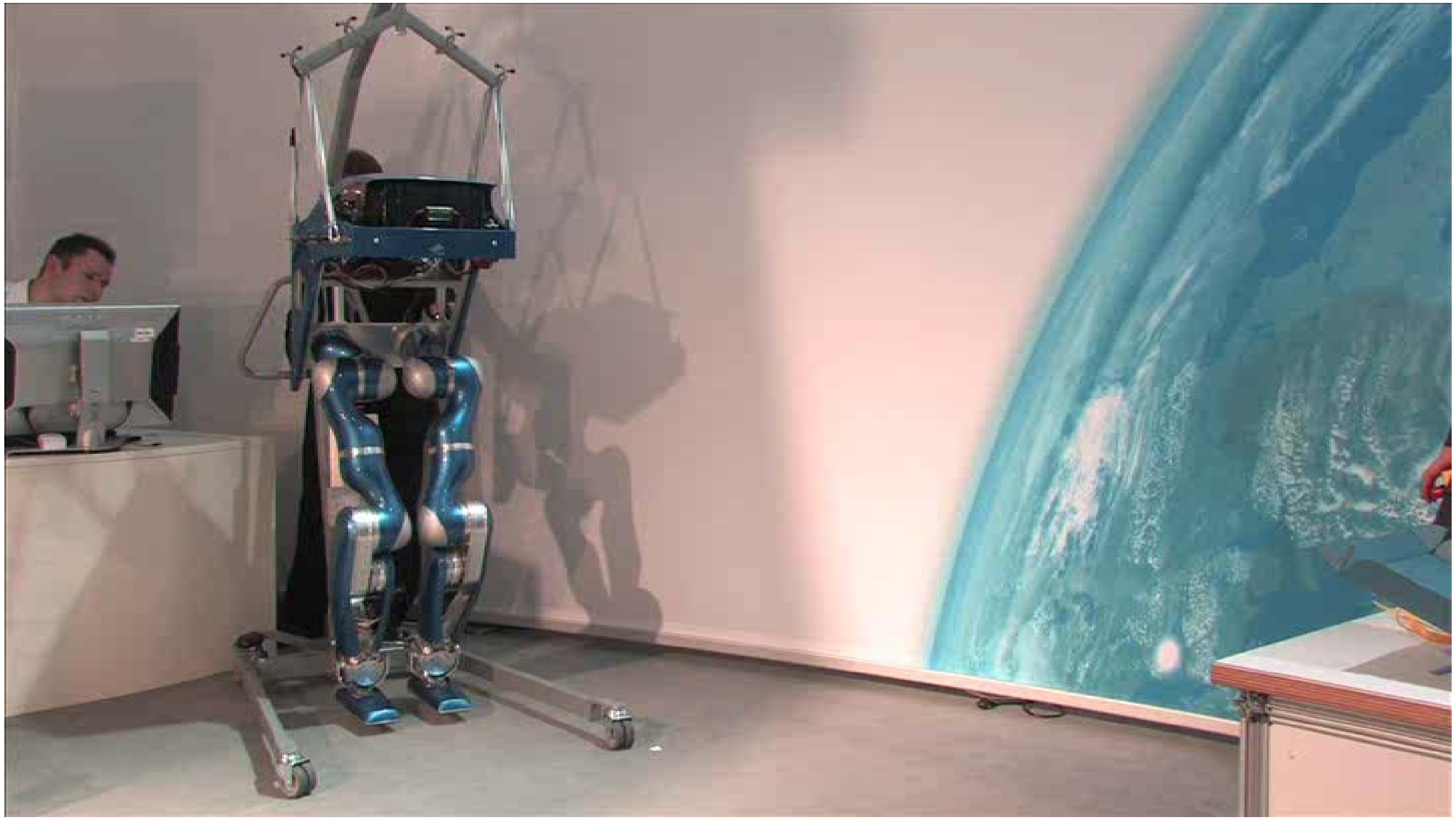
DLR-Biped
(2010-2012)

TORO, preliminary version
(2012)



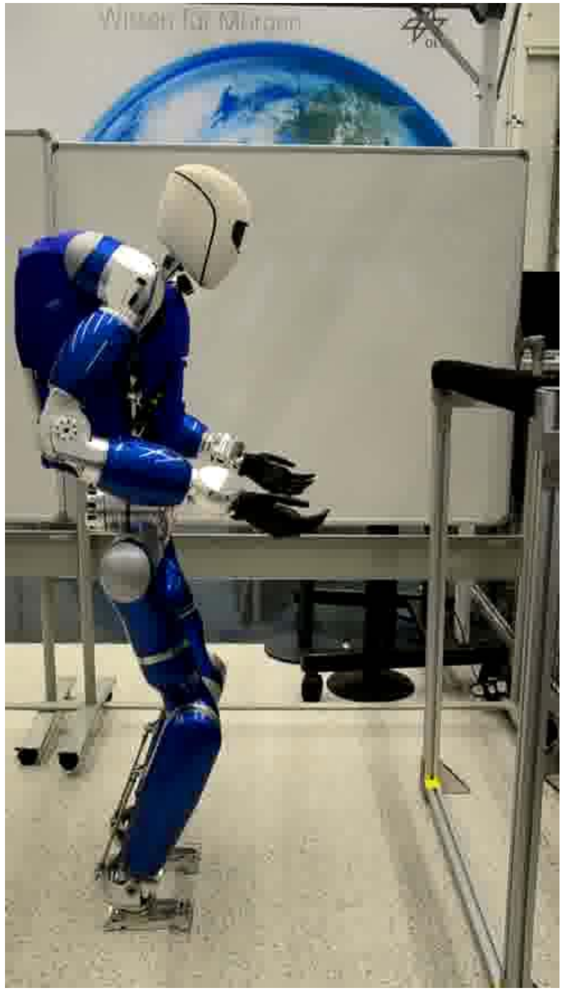
TORO (2013)
TOrque controlled
humanoid RObot

First experiments with DLR-Biped



First experiment at Automatica Fair in 06-2010: ZMP preview control [Kajita, 2003]
Current approach: Walking control based on the Capture Point
[Englsberger, Ott, Roa, et al. IROS 2011]

TORO



Overview

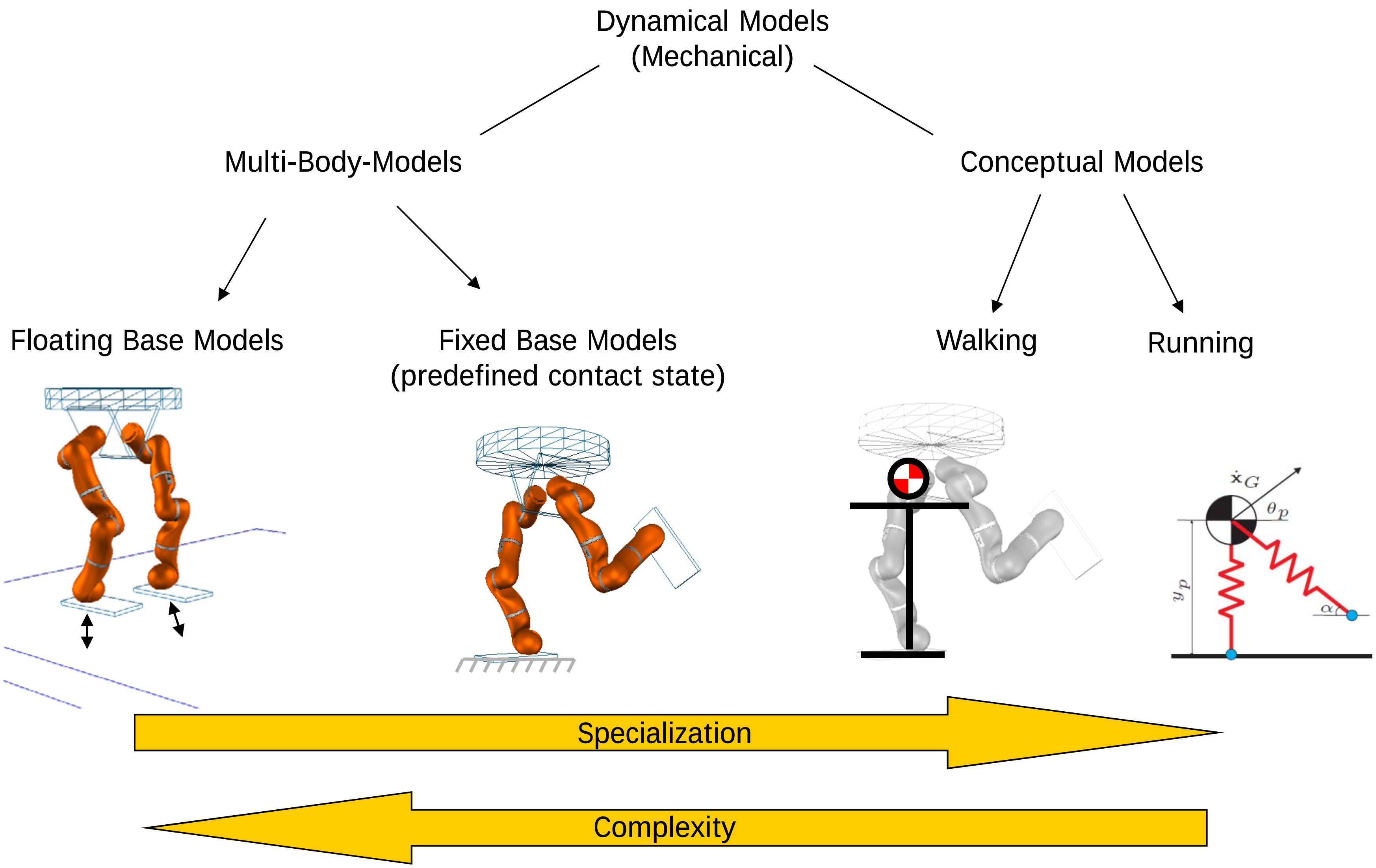
Part I: Modeling

- Multibody dynamics
- ZMP
- Simplified models for control
- Capture Point
- Centroidal Moment Pivot

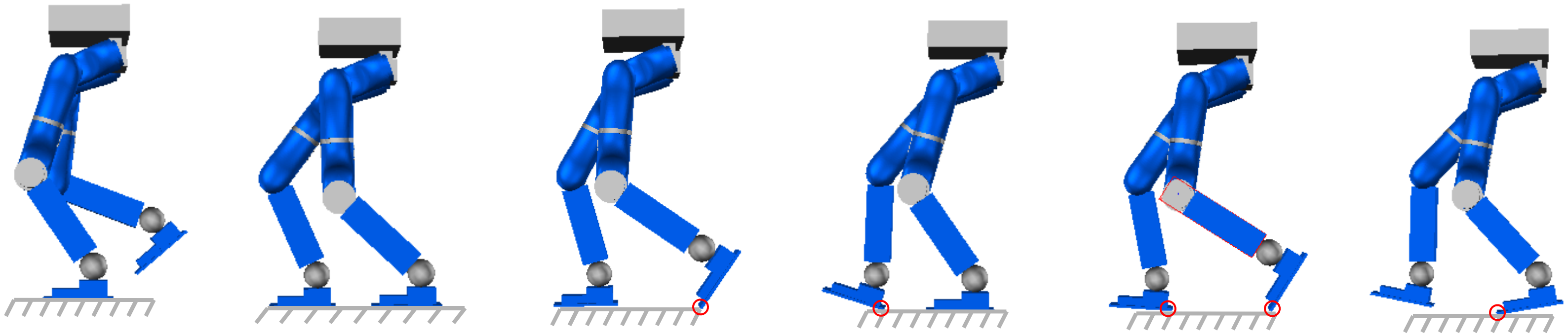
Part II: Balancing

Part III: Walking Control

Models of Legged (Humanoid) Robots



Free-Floating vs. Fixed Base Models



single support
serial kin. chain

double support
over-contrained

underactuated

Fixed base models

In each contact state the model is different:

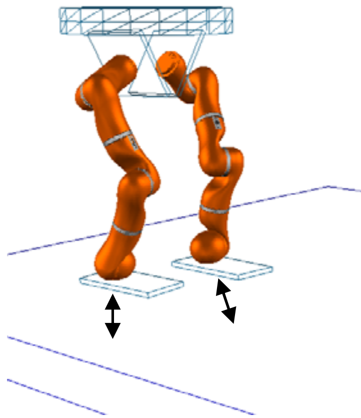
- Single support (right, left)
- Double support
- Heel Off
- Toe Touch Down
- ...

Transition between contact states

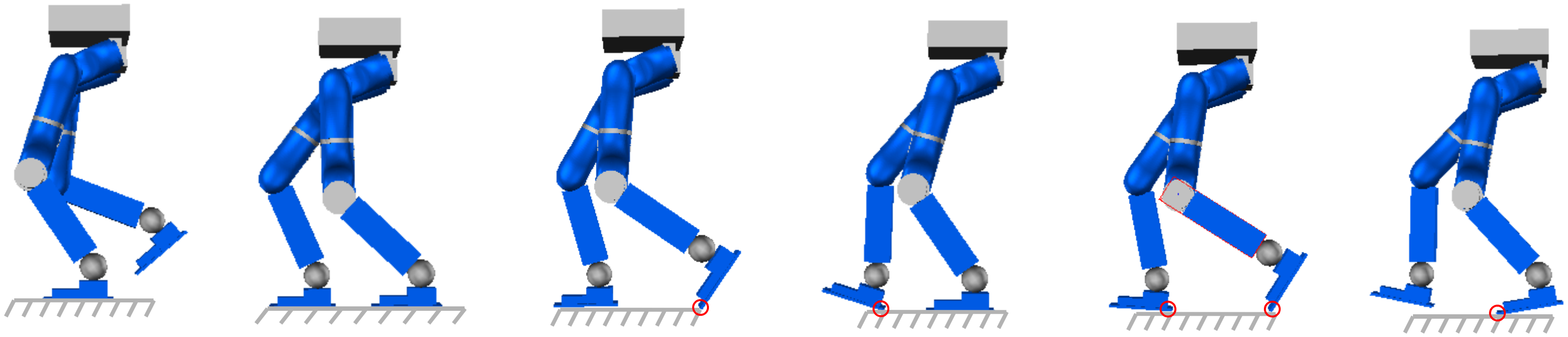
Free-floating model

Components:

- Lagrangian dynamics
- Constraints due to contact forces
- Transition equations (impacts)



Free-Floating vs. Fixed Base Models



single support
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Free-floating model

Components:
- Lagrangian dynamics
- Constraints due to contact forces
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Fixed base models
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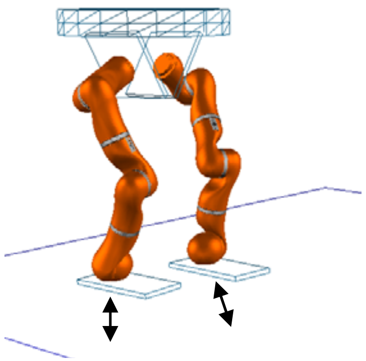
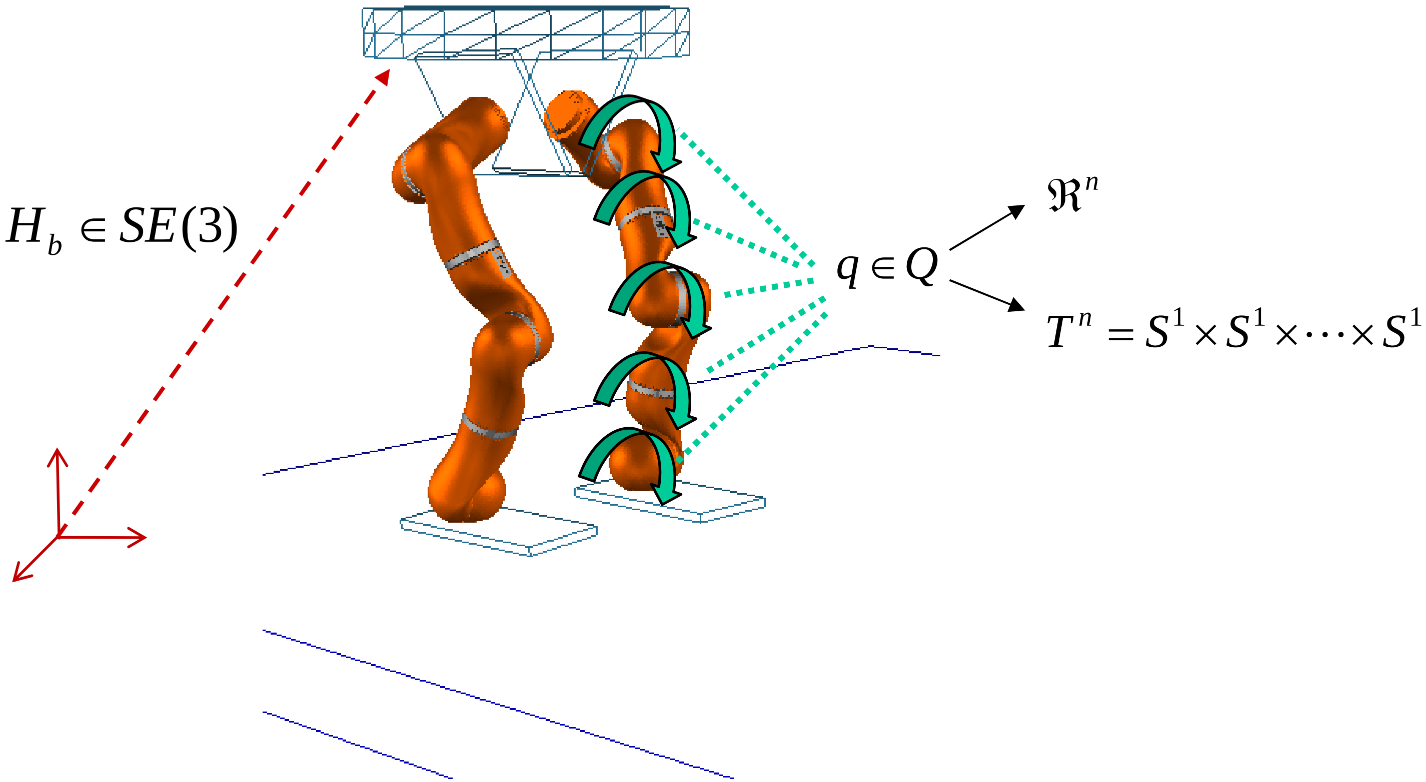


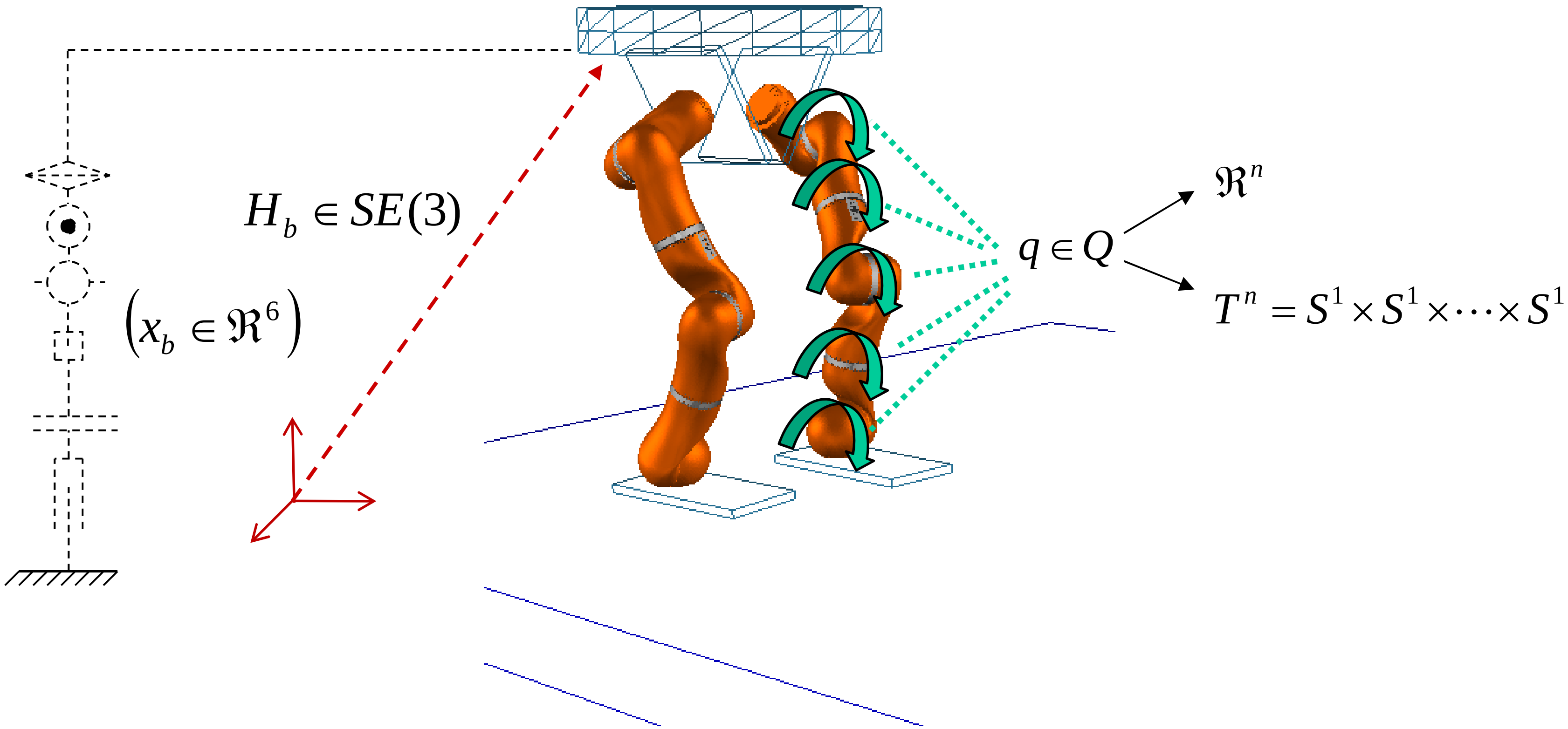
Planning & control must ensure that the considered contact state is valid!
→ ground reaction force must fulfill constraints

Configuration Space



Configuration Space: $Q \times SE(3)$

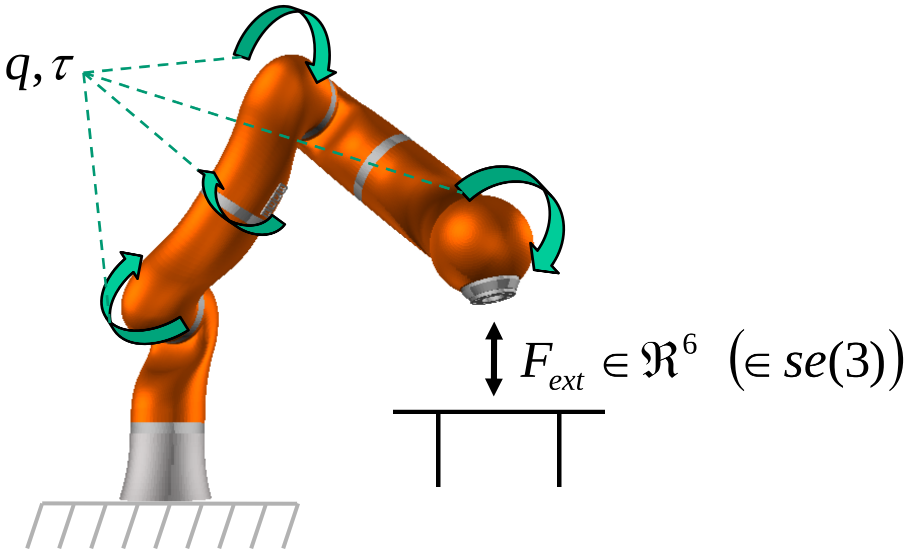
Configuration Space



Configuration Space: $Q \times SE(3)$

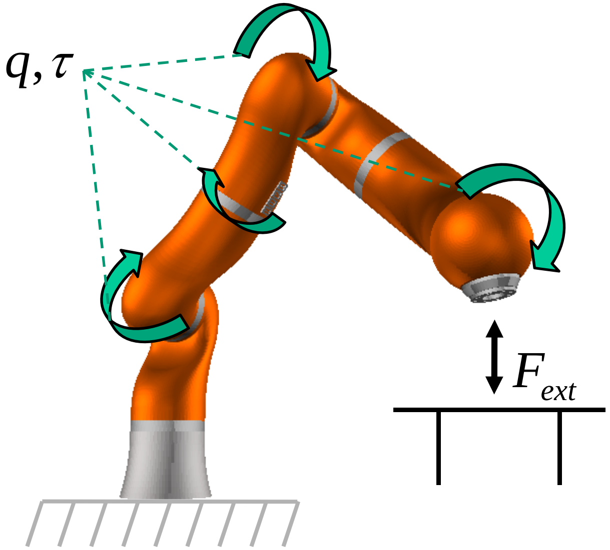
Using local coordinates: $\Re^{6+n}$

Modeling



$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau + J(q)^T F_{ext}$$

Modeling



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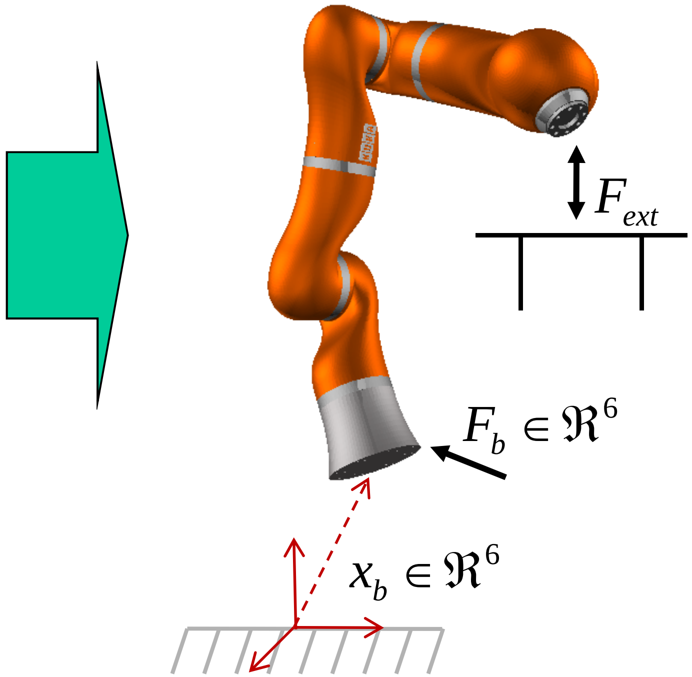
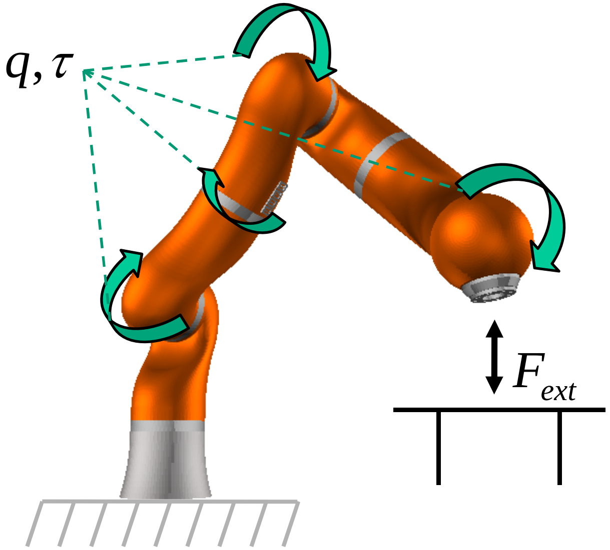
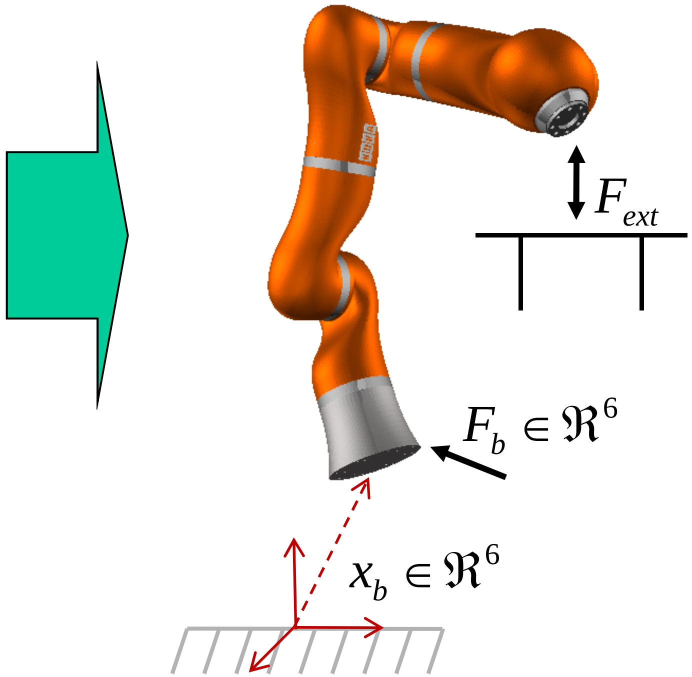


Modeling

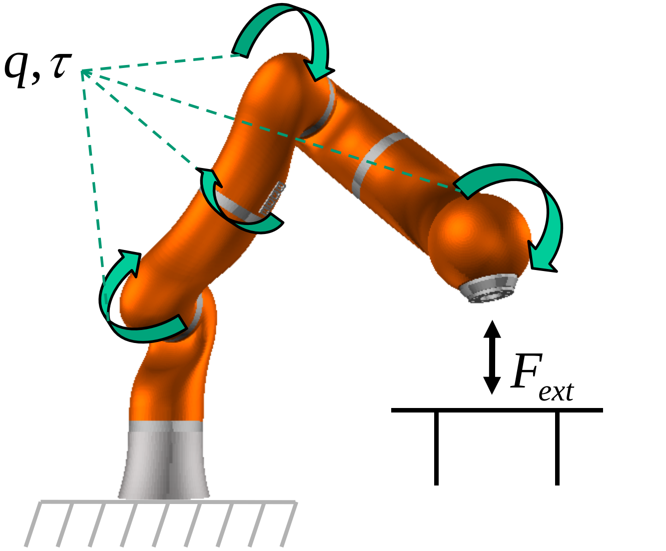




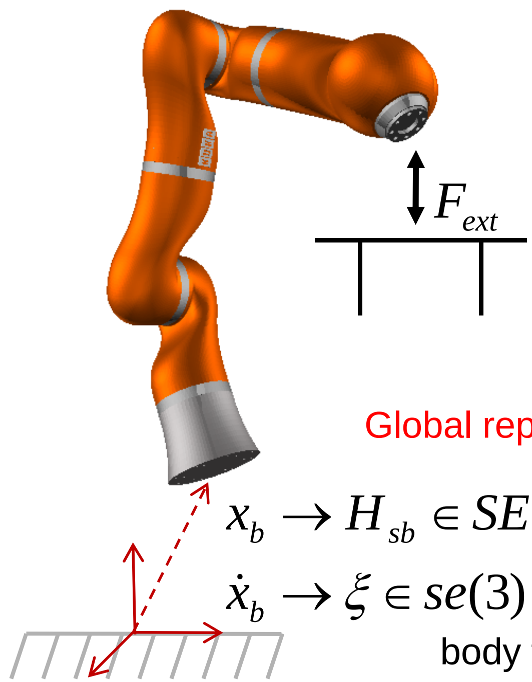
$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau + J(q)^T F_{ext}$$

$$\begin{bmatrix} M_x(q) & M_{xq}(q) \\ M_{qx}(q) & M(q) \end{bmatrix} \begin{pmatrix} \ddot{x}_b \\ \ddot{q} \end{pmatrix} + \overline{C}(q,\dot{x}_b,\dot{q}) \begin{pmatrix} \dot{x}_b \\ \dot{q} \end{pmatrix} + \overline{g}(x_b,q) = \begin{pmatrix} F_b \\ \tau \end{pmatrix} + \begin{bmatrix} J_b(q)^T \\ J(q)^T \end{bmatrix} F_{ext}$$

Modeling



$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau + J(q)^T F_{ext}$$



Global representation!

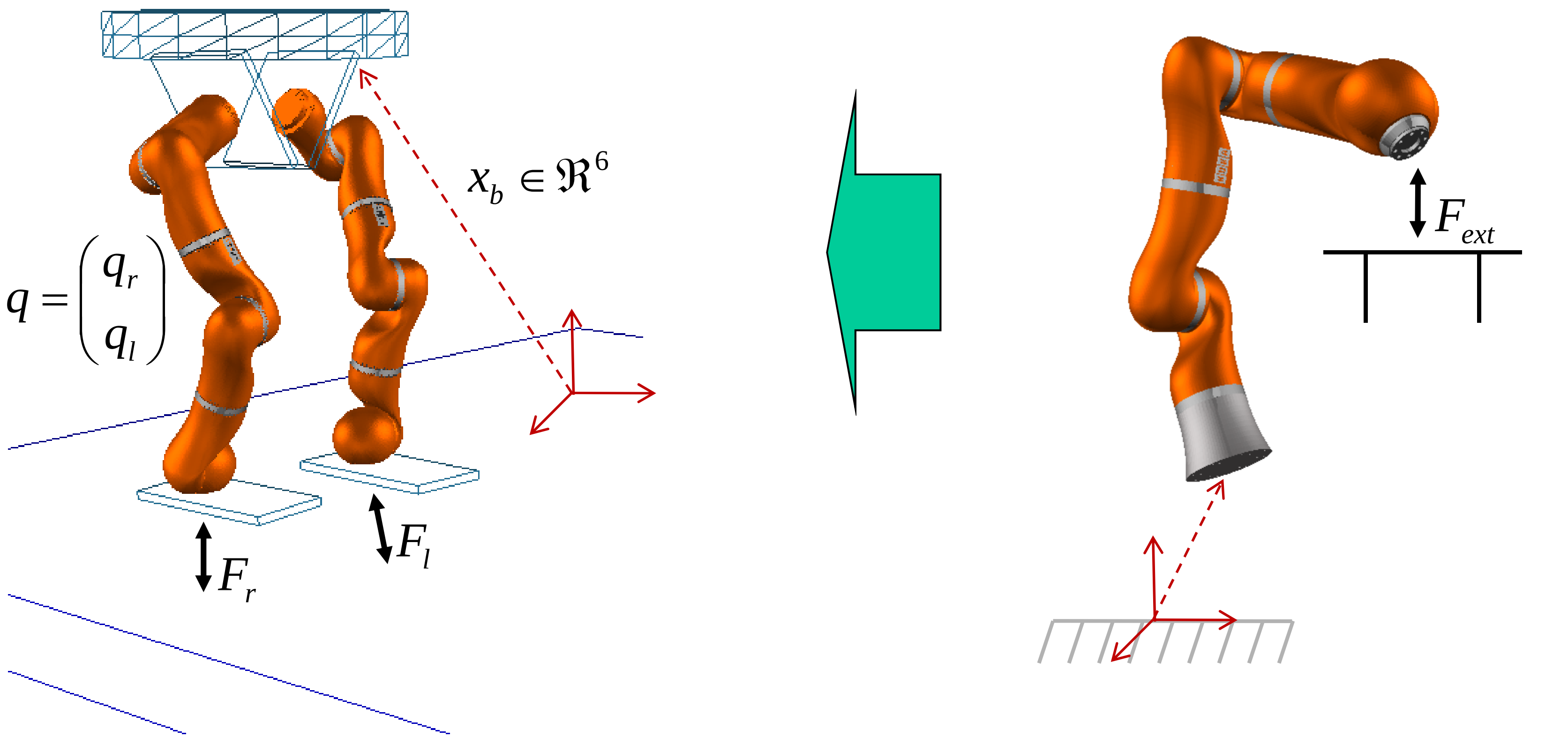
$x_b \rightarrow H_{sb} \in SE(3)$

$\dot{x}_b \rightarrow \xi \in se(3)$

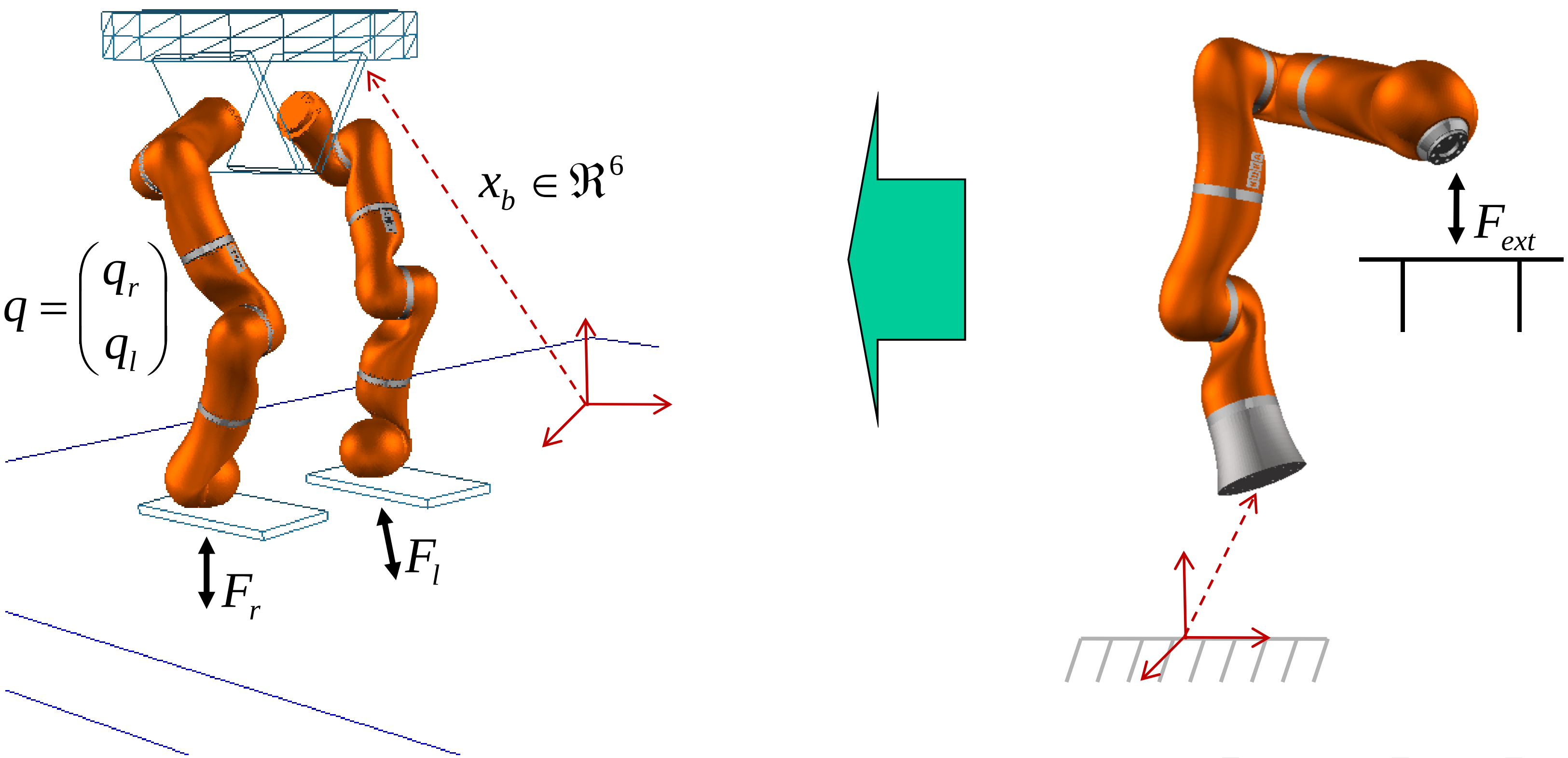
body twist

$$\begin{bmatrix} M_b(q) & M_{bq}(q) \\ M_{qb}(q) & M(q) \end{bmatrix} \begin{pmatrix} \dot{\xi} \\ \ddot{q} \end{pmatrix} + \overline{C}_b(q,\xi,\dot{q}) \begin{pmatrix} \xi \\ \dot{q} \end{pmatrix} + \overline{g}_b(H_{sb}, q) = \begin{pmatrix} W_b \\ \tau \end{pmatrix} + \begin{bmatrix} Ad_{bt}(q)^T \\ J(q)^T \end{bmatrix} F_{ext}$$

Modeling

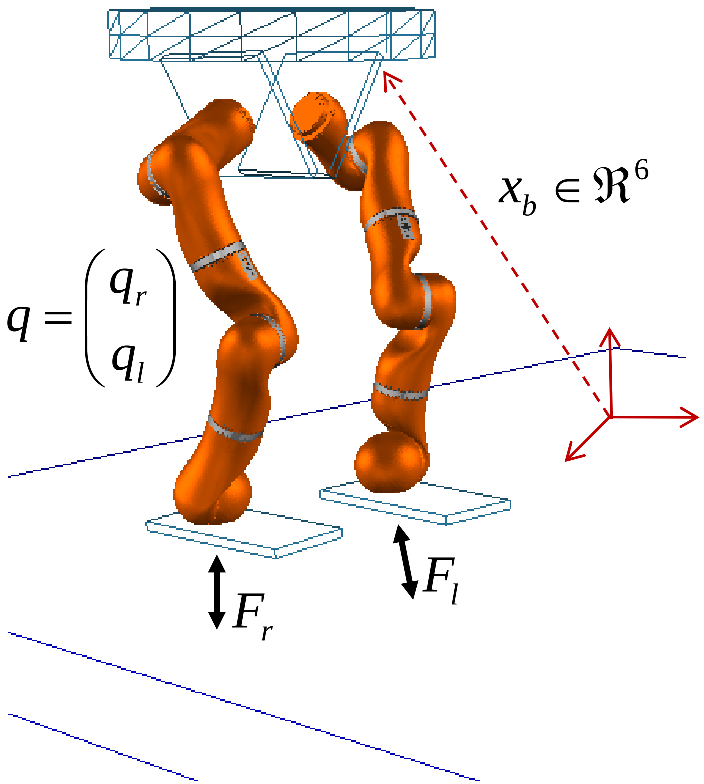


Modeling



$$\begin{bmatrix} M_x(q) & M_{xq}(q) \\ M_{qx}(q) & M(q) \end{bmatrix} \begin{pmatrix} \ddot{x}_b \\ \ddot{q} \end{pmatrix} + \overline{C}(q, \dot{x}_b, \dot{q}) \begin{pmatrix} \dot{x}_b \\ \dot{q} \end{pmatrix} + \overline{g}(x_b, q) = \begin{pmatrix} 0 \\ \tau \end{pmatrix} + \begin{bmatrix} J_{br}(q)^T \\ J_r(q)^T \\ 0 \end{bmatrix} F_r + \begin{bmatrix} J_{bl}(q)^T \\ 0 \\ J_l(q)^T \end{bmatrix} F_l$$

Bipedal Robot Model



Properties for control:

- Underactuated
- Varying unilateral constraints (single support, double support, edge contact)
- Constraints on the state & control

$$\begin{bmatrix} M_x(q) & M_{xq}(q) \\ M_{qx}(q) & M(q) \end{bmatrix}\begin{pmatrix} \ddot{x}_b \\ \ddot{q} \end{pmatrix} + \overline{C}(q, \dot{x}_b, \dot{q})\begin{pmatrix} \dot{x}_b \\ \dot{q} \end{pmatrix} + \overline{g}(x_b, q) = \begin{pmatrix} 0 \\ \tau \end{pmatrix} + \begin{bmatrix} J_{br}(q)^T \\ J_r(q)^T \\ 0 \end{bmatrix} F_r + \begin{bmatrix} J_{bl}(q)^T \\ 0 \\ J_l(q)^T \end{bmatrix} F_l$$

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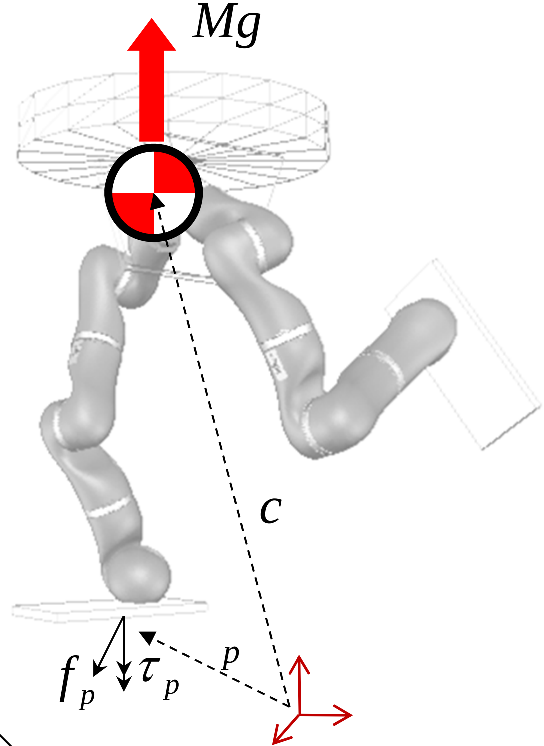
Bipedal Robot Model

DLR

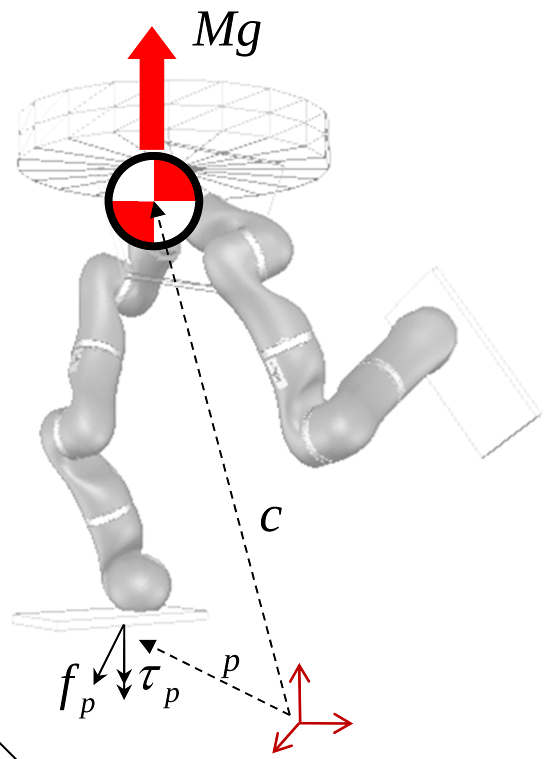


$$\begin{bmatrix} M & 0 \\ 0 & \hat{M}(q) \end{bmatrix} \begin{pmatrix} \ddot{c} \\ \ddot{\hat{q}} \end{pmatrix} + \begin{bmatrix} 0 \\ \hat{C}(\hat{q}, \dot{\hat{q}}) \end{bmatrix} + \begin{bmatrix} -Mg \\ 0 \end{bmatrix} = \begin{pmatrix} 0 \\ u \end{pmatrix} - \sum_{i=r,l} \begin{bmatrix} I & 0 \\ J_i(\hat{q})^T \end{bmatrix} F_i$$

$$\begin{pmatrix} \varepsilon \\ q \end{pmatrix} \qquad (c, \varepsilon) \Leftarrow x_b \qquad \begin{pmatrix} \tau_p \\ \tau \end{pmatrix}$$

$$\begin{bmatrix} M_x(q) & M_{xq}(q) \\ M_{qx}(q) & M(q) \end{bmatrix} \begin{pmatrix} \ddot{x}_b \\ \ddot{q} \end{pmatrix} + \bar{C}(q, \dot{x}_b, \dot{q}) \begin{pmatrix} \dot{x}_b \\ \dot{q} \end{pmatrix} + \bar{g}(x_b, q) = \begin{pmatrix} 0 \\ \tau \end{pmatrix} + \begin{bmatrix} J_{br}(q)^T \\ J_r(q)^T \\ 0 \end{bmatrix} F_r + \begin{bmatrix} J_{bl}(q)^T \\ 0 \\ J_l(q)^T \end{bmatrix} F_l$$

Bipedal Robot Model



Conservation of momentum:

$$M\ddot{c} = Mg - \sum_{i=r,l} f_i$$

$$\begin{bmatrix} M & 0 \\ 0 & \hat{M}(q) \end{bmatrix} \begin{pmatrix} \ddot{c} \\ \ddot{\hat{q}} \end{pmatrix} + \begin{bmatrix} 0 \\ \hat{C}(\hat{q}, \dot{\hat{q}}) \end{bmatrix} + \begin{bmatrix} -Mg \\ 0 \end{bmatrix} = \begin{pmatrix} 0 \\ u \end{pmatrix} - \sum_{i=r,l} \begin{bmatrix} I & 0 \\ J_i(\hat{q})^T \end{bmatrix} F_i$$

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Bipedal Robot Model

Conservation of angular momentum:

$$\dot{L} = c \times Mg + \sum \tau_i$$

Conservation of momentum:

$$M\ddot{c} = Mg - \sum_{i=r,l} f_i$$

$$\begin{bmatrix} M & 0 \\ 0 & \hat{M}(q) \end{bmatrix} \begin{pmatrix} \ddot{c} \\ \ddot{\hat{q}} \end{pmatrix} + \begin{bmatrix} 0 \\ \hat{C}(\hat{q}, \dot{\hat{q}}) \end{bmatrix} + \begin{bmatrix} -Mg \\ 0 \end{bmatrix} = \begin{pmatrix} 0 \\ u \end{pmatrix} - \sum_{i=r,l} \begin{bmatrix} I & 0 \\ J_i(\hat{q})^T \end{bmatrix} F_i$$

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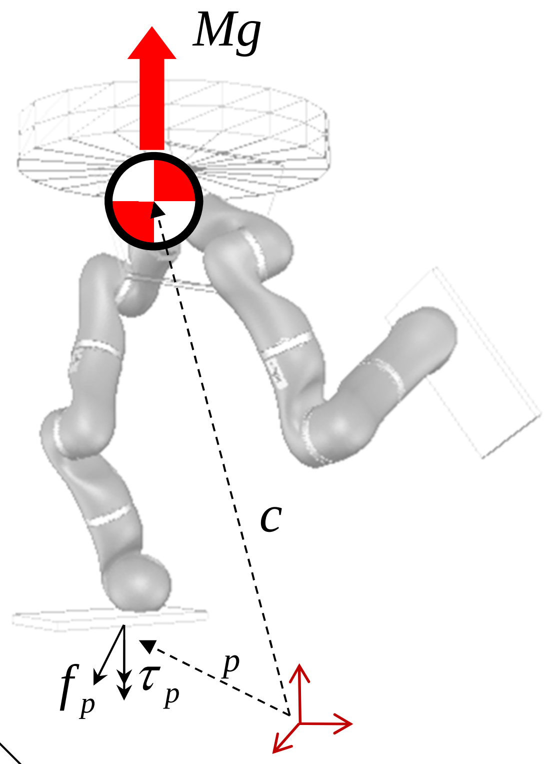

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Bipedal Robot Model

On a flat ground:

$$\tau_p = \dot{L} - c \times Mg - p \times (M\ddot{c} - Mg)$$

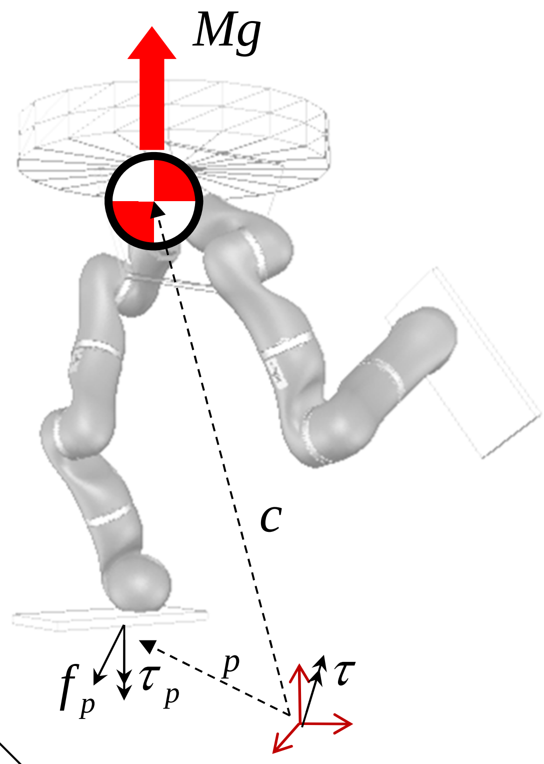
Conservation of angular momentum:

$$\dot{L} = c \times Mg + \sum \tau_i$$

Conservation of momentum:

$$M\ddot{c} = Mg - \sum_{i=r,l} f_i$$

$$\tau = p \times f_p + \tau_p$$



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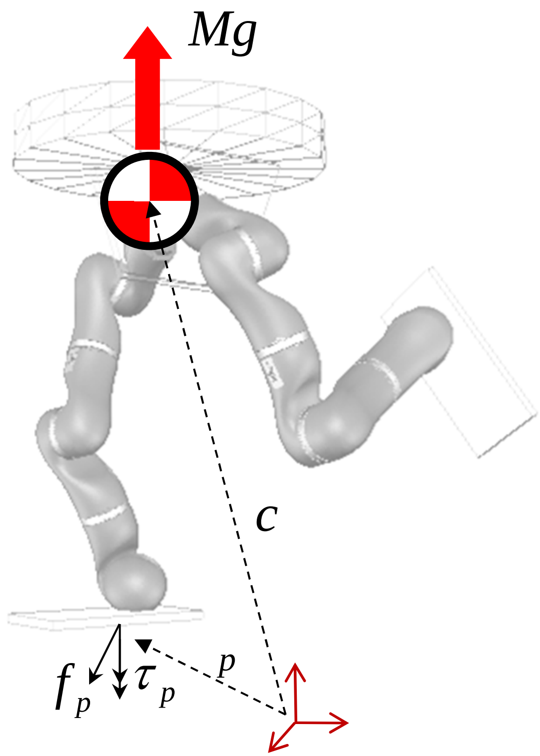
Bipedal Robot Model

On a flat ground:

$$\boxed{\tau_p = \dot{L} - c \times Mg - p \times (M\ddot{c} - Mg)}$$



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(c, ε)

x_b

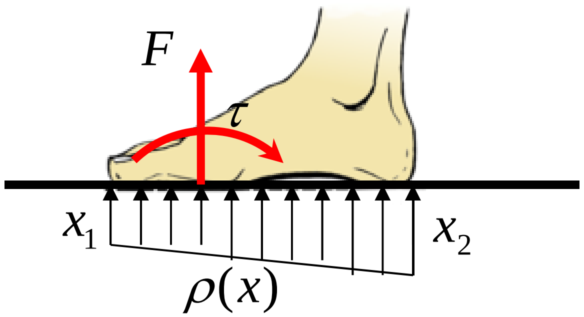
$$\begin{bmatrix} M_x(q) & M_{xq}(q) \\ M_{qx}(q) & M(q) \end{bmatrix} \begin{pmatrix} \ddot{x}_b \\ \ddot{q} \end{pmatrix} + \overline{C}(q, \dot{x}_b, \dot{q}) \begin{pmatrix} \dot{x}_b \\ \dot{q} \end{pmatrix} + \overline{g}(x_b, q) = \begin{pmatrix} 0 \\ \tau \end{pmatrix} + \overbrace{\begin{bmatrix} J_{br}(q)^T \\ J_r(q)^T \\ 0 \end{bmatrix} F_r + \begin{bmatrix} J_{bl}(q)^T \\ 0 \\ J_l(q)^T \end{bmatrix} F_l}$$

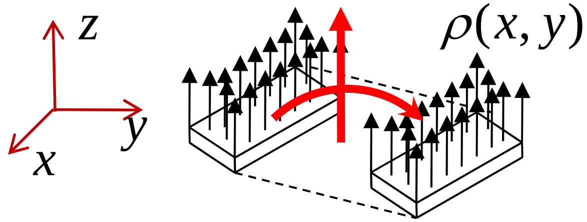
Zero Moment Point

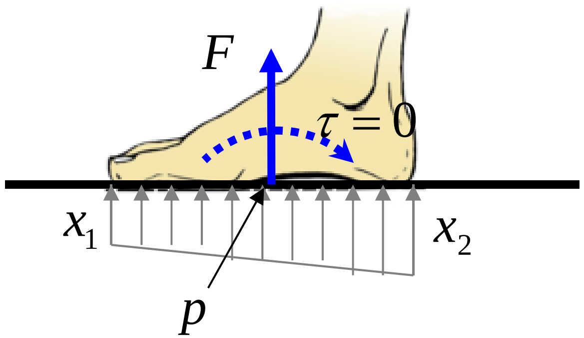
Zero Moment Point

[Vukobratovic and Stepanenko,1972]

ZMP as a ground reference point: Distributed ground reaction force under the supporting foot can be replaced by a single force acting at the ZMP.







ZMP = CoP (Center of Pressure)

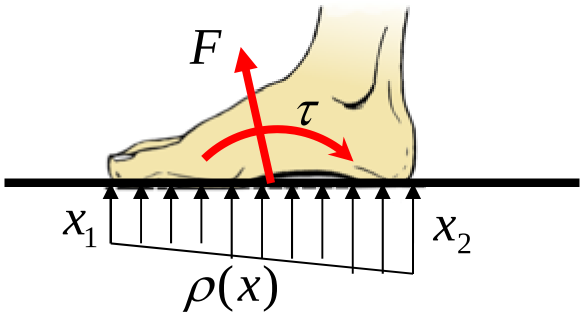
$\rho(x, y) > 0 \quad \Rightarrow \quad p$ in convex hull of the support polygon.

Some facts about the ZMP

- Can ZMP leave the support polygon? → NO
- Can ZMP location be used as a stability criterion → NO

- If ZMP reaches the border of the support polygon → foot rotation possible.

- ZMP is defined on flat contact (no uneven surface).
- ZMP gives no information about sliding.



First usage of the ZMP

- Motion of the legs is predefined.
- Upper body controls the ZMP in the center of the supporting foot

  → ensure proper foot contact during walking

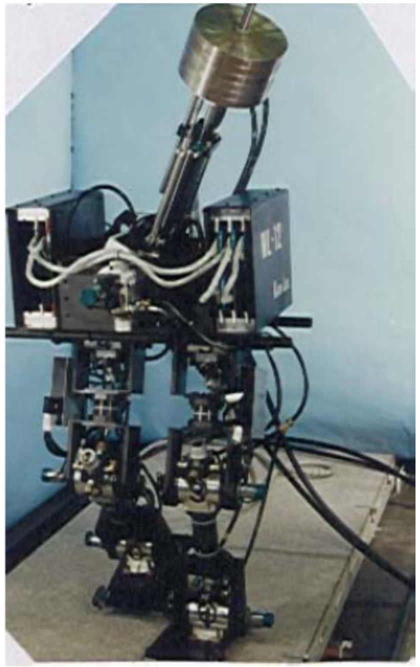


Figure 9
WL-12 (1986)

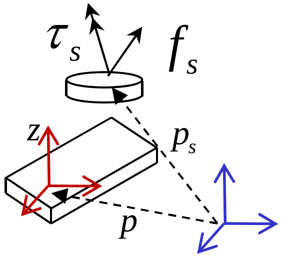
How to obtain the ZMP?

Measurement
e.g. by Force/Torque Sensor

Dynamics Computation

How to obtain the ZMP?

Measurement
e.g. by Force/Torque Sensor



$$\tau(p) = (p_s - p) \times f_s + \tau_s$$

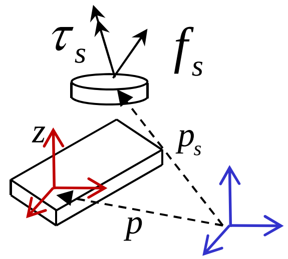
$$p_x = \frac{-\tau_y - (p_{sz} - p_z) f_x + p_{sx} f_z}{f_z}$$

$$p_y = \frac{\tau_x - (p_{sz} - p_z) f_y + p_{sy} f_z}{f_z}$$

Dynamics Computation

How to obtain the ZMP?

Measurement
e.g. by Force/Torque Sensor



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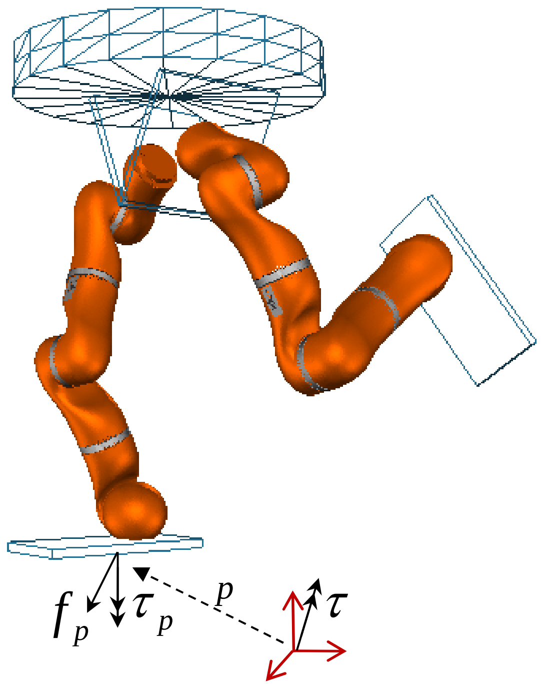
Dynamics Computation



$$\tau = p \times f_p + \tau_p$$

How to obtain the ZMP?

$$\dot{P} = Mg + f$$

$$\dot{L} = c \times Mg + \tau$$

$$\tau_p = \dot{L} - c \times Mg - p \times (\dot{P} - Mg)$$

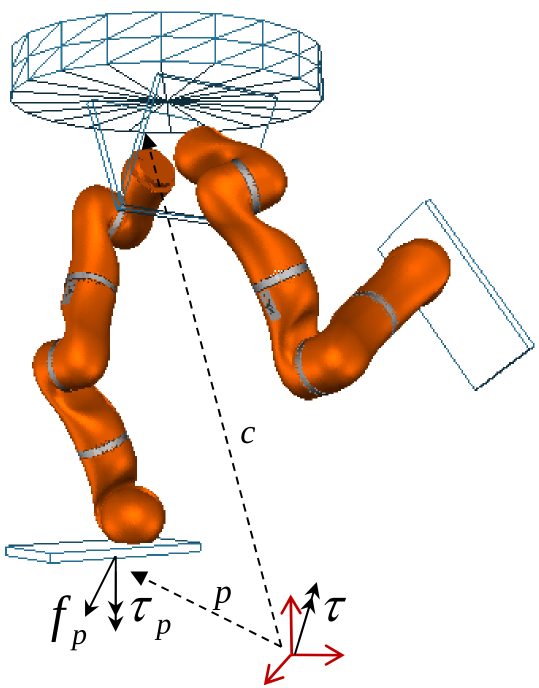
$$\tau_{px} = 0$$

$$\tau_{py} = 0$$

$$p_x = \frac{Mgc_x + p_z \dot{P}_x - \dot{L}_y}{Mg + \dot{P}_z}$$

$$p_y = \frac{Mgc_y + p_z \dot{P}_y + \dot{L}_x}{Mg + \dot{P}_z}$$

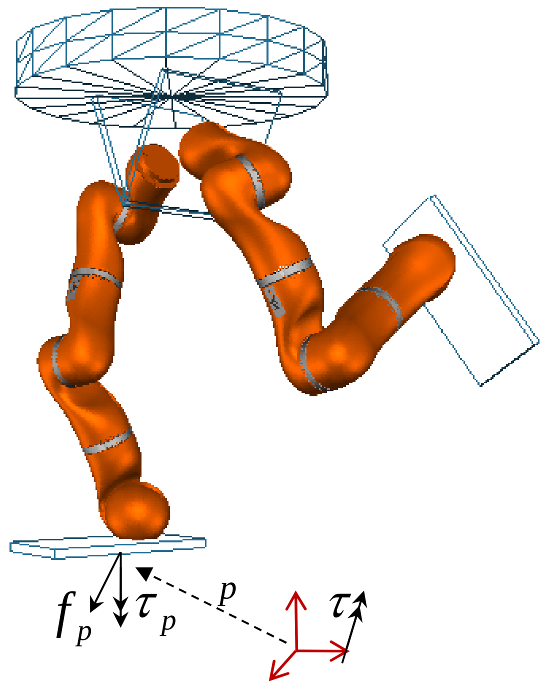
Dynamics Computation



$$\tau = p \times f_p + \tau_p$$

A simplified walking model based on the ZMP

Mass concentrated model



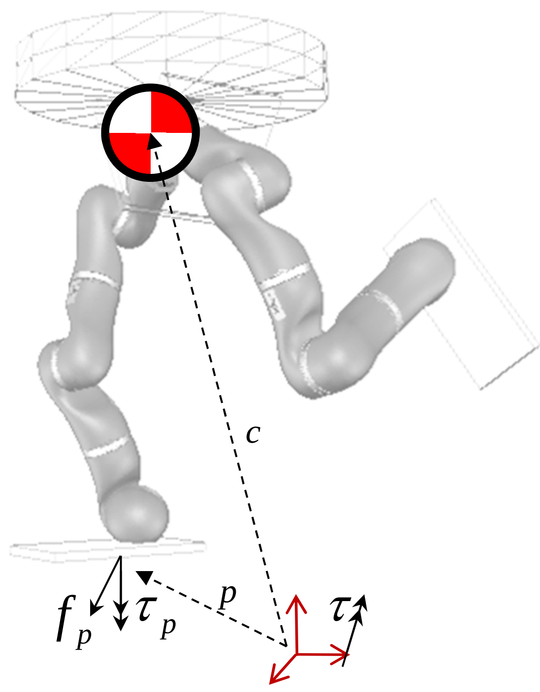
$$\tau = p \times f_p + \tau_p$$

Mass concentrated model

$$P = M\dot{c}$$

$$L = c \times M\dot{c}$$

$$p_z = 0$$



$$\tau = p \times f_p + \tau_p$$

Mass concentrated model

$$p_x = \frac{Mgc_x + p_z\dot{P}_x - \dot{L}_y}{Mg + \dot{P}_z}$$

$$p_y = \frac{Mgc_y + p_z\dot{P}_y + \dot{L}_x}{Mg + \dot{P}_z}$$

$$p_x = c_x - \frac{c_z\ddot{c}_x}{g + \ddot{c}_z}$$

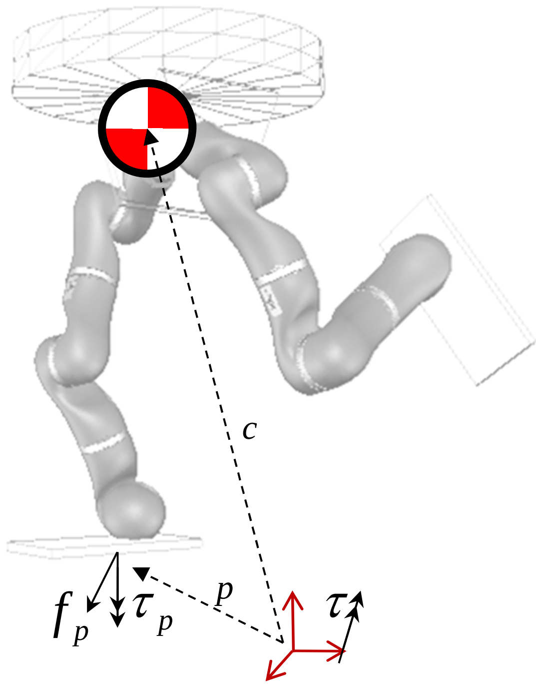
$$p_y = c_y - \frac{c_z\ddot{c}_y}{g + \ddot{c}_z}$$

ZMP of a mass concentrated model

$$P = M\dot{c}$$

$$L = c \times M\dot{c}$$

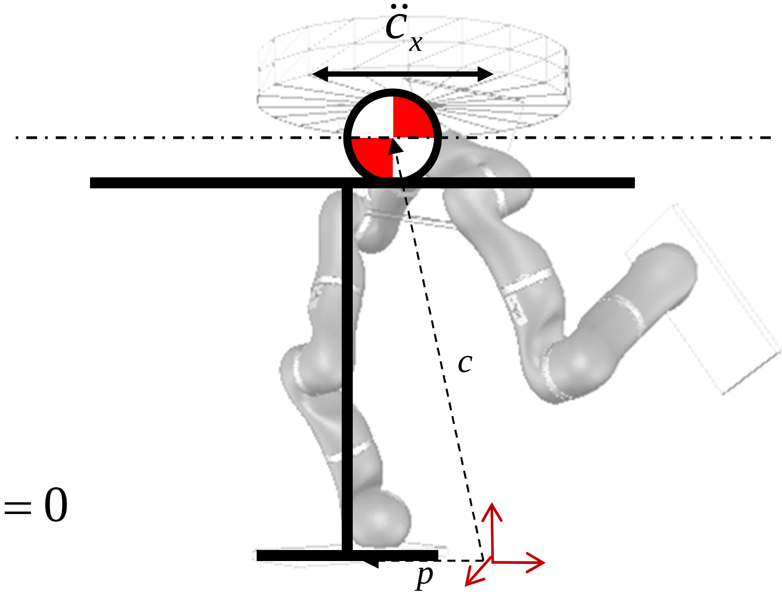
$$p_z = 0$$



$$\tau = p \times f_p + \tau_p$$

Mass concentrated model

Cart-Table Model [Kajita]



$$p_x = c_x - \frac{c_z \ddot{c}_x}{g + \ddot{c}_z}$$

$$p_y = c_y - \frac{c_z \ddot{c}_y}{g + \ddot{c}_z}$$

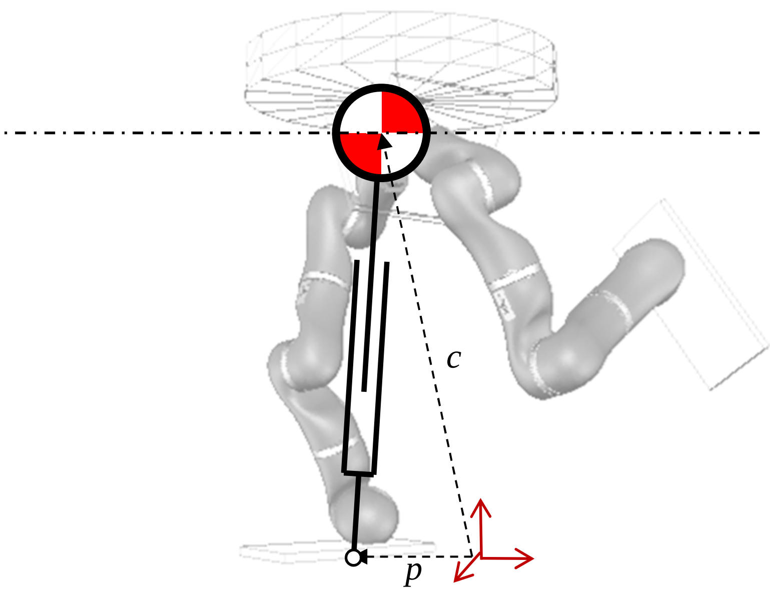
$$\ddot{c}_z = \dot{c}_z = 0$$

$$\boxed{p_x = c_x - \frac{c_z \ddot{c}_x}{g}}$$

$$p_x \leftarrow \ddot{c}_x$$

Mass concentrated model

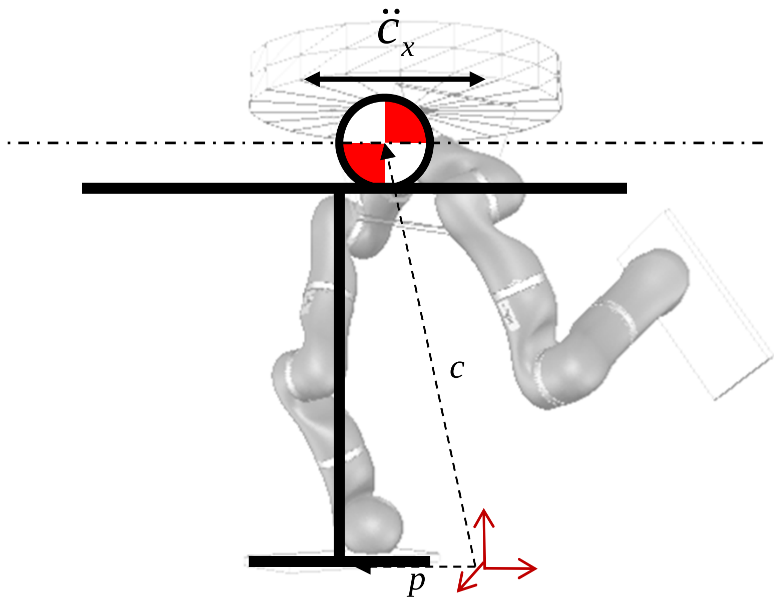
Linear Inverted Pendulum Model [Sugihara]



$$\ddot{c}_x = \frac{g}{c_z}(c_x - p_x)$$

$$\ddot{c}_x \leftarrow p_x$$

Cart-Table Model [Kajita]



$$p_x = c_x - \frac{c_z \ddot{c}_x}{g}$$

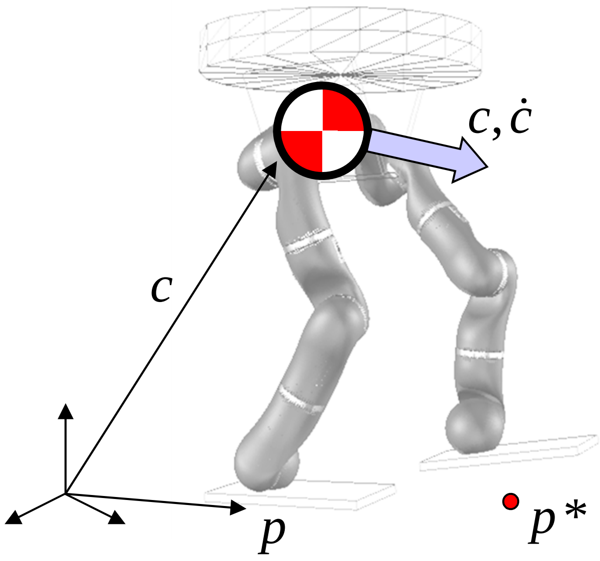
$$p_x \leftarrow \ddot{c}_x$$

Capture Point
(Extrapolated Center of Mass)
((Divergent Component of Motion))

Capture Point

Definition of the "Capture Point" (Pratt 2006, Hof 2008):

Point to step in order to bring the robot to stand.



Can be computed exactly for simple models, e.g. linear inverted pendulum model:

$$\ddot{x} = \omega^2 \left(x - \underset{\text{ZMP}}{\underset{\uparrow}{p}}\right) \qquad \omega = \sqrt{\frac{g}{c_z}}$$

Computation of the Capture Point:

$$p = \text{const} \longrightarrow x(t) = \cosh(\omega t)x(0) + \sinh(\omega t)\frac{\dot{x}(0)}{\omega} + (1 - \cosh(\omega t))p$$

$$x(t \rightarrow \infty) = p$$

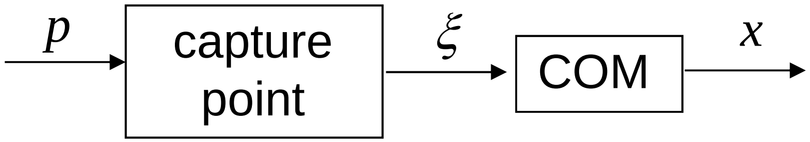
$$\boxed{p* = x_0 + \frac{\dot{x}_0}{\omega}}$$

Capture Point Dynamics

Coordinate transformation: $(x, \dot{x}) \rightarrow (x, \xi)$ $\qquad \xi = x + \dfrac{\dot{x}}{\omega}$

$$\ddot{x} = \omega^2 (x - p) \quad \Rightarrow \quad \boxed{\begin{aligned} \dot{x} &= -\omega x + \omega \xi \\ \dot{\xi} &= \omega \xi - \omega p \end{aligned}}$$

System structure: Cascaded system



open loop unstable

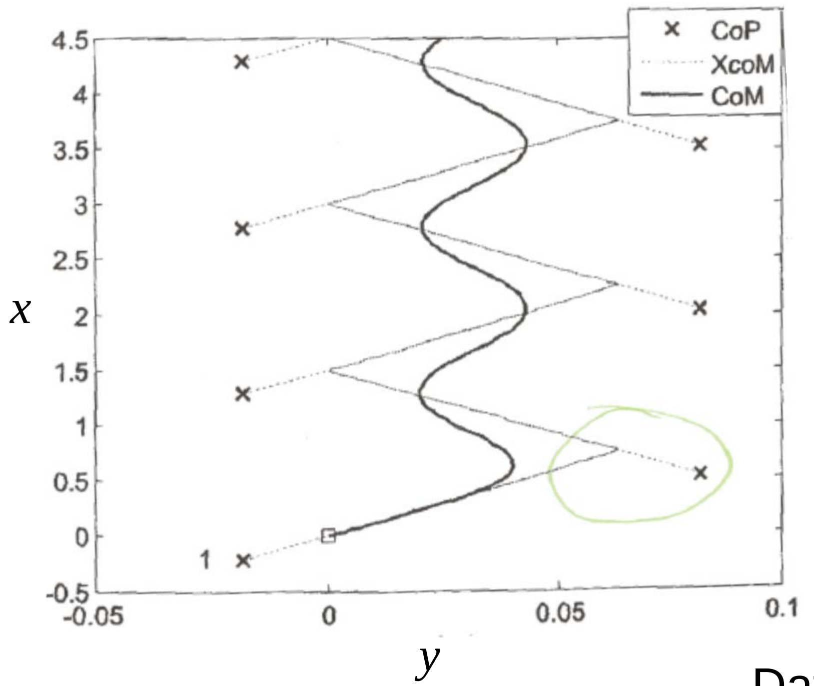
exp. stable

Dual use of the capture point for robotics

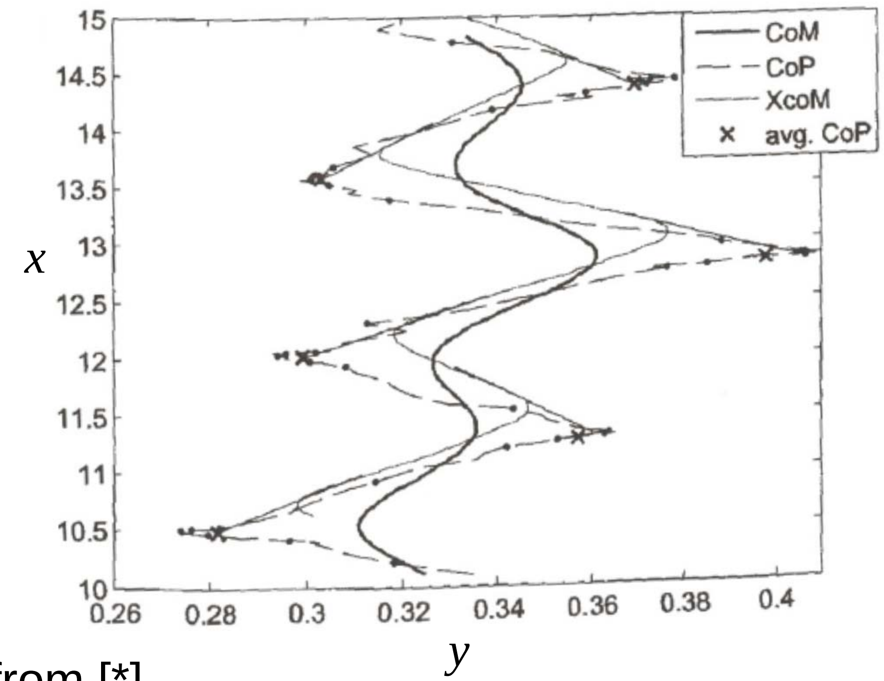
1. step planning
2. control

Capture Point in Human Measurements

Linear Inverted Pendulum



Human



Data from [*]

[*] Hof, *The extrapolated center of mass concept suggests a simple control of balance in walking*, Human Movement Science 27, pp.112-125, 2008.

Centroidal Moment Pivot

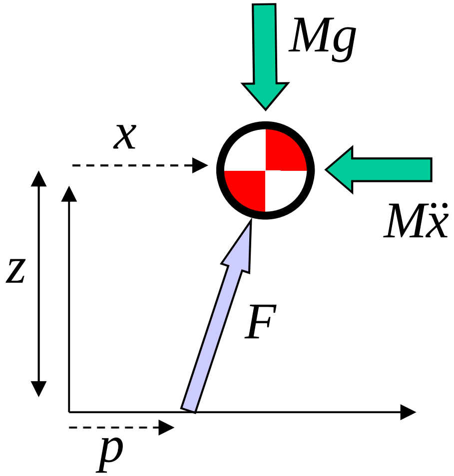
Centroidal Moment Pivot

- Observation in human data: For normal level-ground walking, the human body‘s angular momentum (and the angular excursions) about the COM remains small through the gait cycle.

- The centroidal moment pivot was introduced as a ground reference point to address the effects of angular momentum about the COM in connection with postural balance strategies.

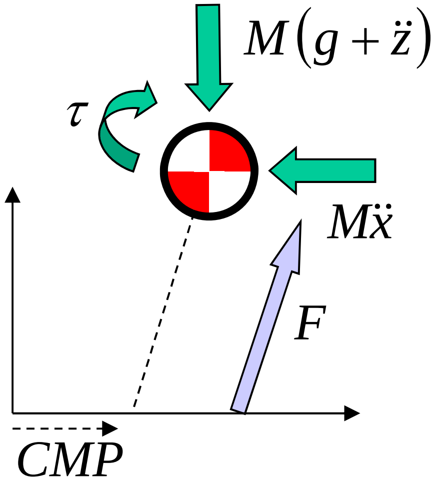
Centroidal Moment Pivot

Forces in the LIP model



$$\ddot{x} = \frac{g}{z}(x - p)$$

Effect of an additional hip torque



$$\ddot{x} = \frac{g + \ddot{z}}{z}(x - p) + \frac{\tau}{Mz}$$

$$CMP = p - \frac{\tau}{F_z}$$

Interpretation

- The distance between CMP and ZMP corresponds to the angular momentum about the COM.
- While the ZMP cannot leave the support polygon (by definition), the CMP can leave it.
- The distance between CMP and the support polygon has been proposed as an indicator which balance strategy should dominate (via ZMP or via angular momentum).

Overview

Part I: Modeling

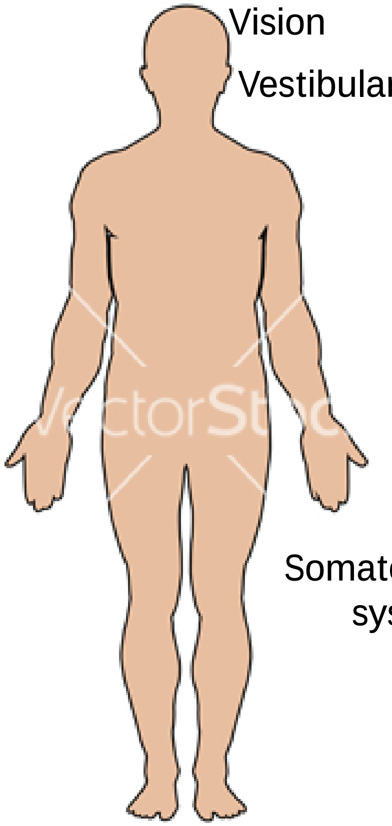
Part II: Balancing

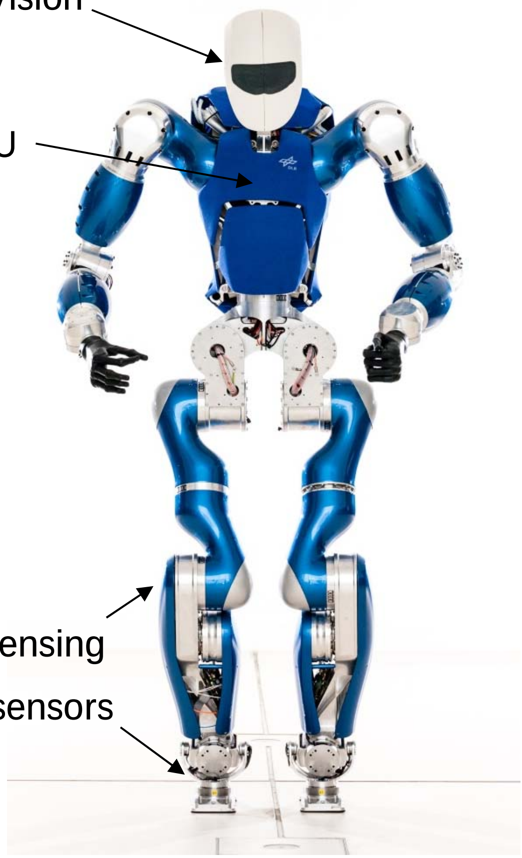
1. Basics
2. ZMP based balancing (concentrated mass model)
3. Torque based balancing (multi body dynamics)

Part III: Walking Control

Humanoid Balance

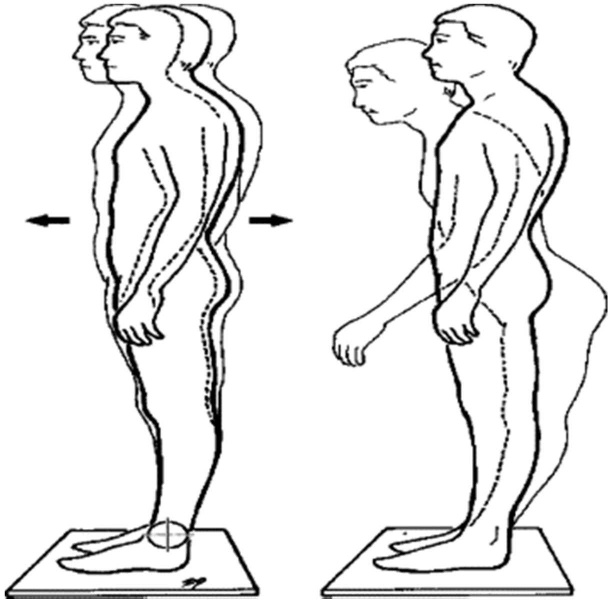
“Balance” is a generic term describing the ability to control the body posture in order to prevent falling.





Humanoid Balance

Strategies for human push recovery:

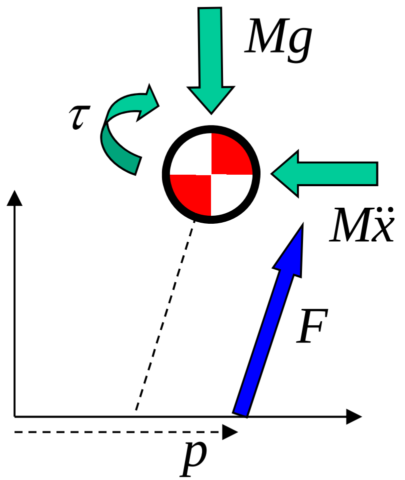
	Small push: Ankle strategy	Medium push: Hip strategy	Large Push: Step strategy
Human			
Robot	force control ZMP control	angular momentum control	

mass concentrated model

Strategies for gait stabilization:

1. Controlling ZMP (constraints!)
2. Angular momentum control
3. Step adaptation

Effect of an additional hip torque



$$\ddot{x} = \frac{g}{z}(x - p) + \frac{\tau}{Mz}$$

Overview

Part I: Modeling

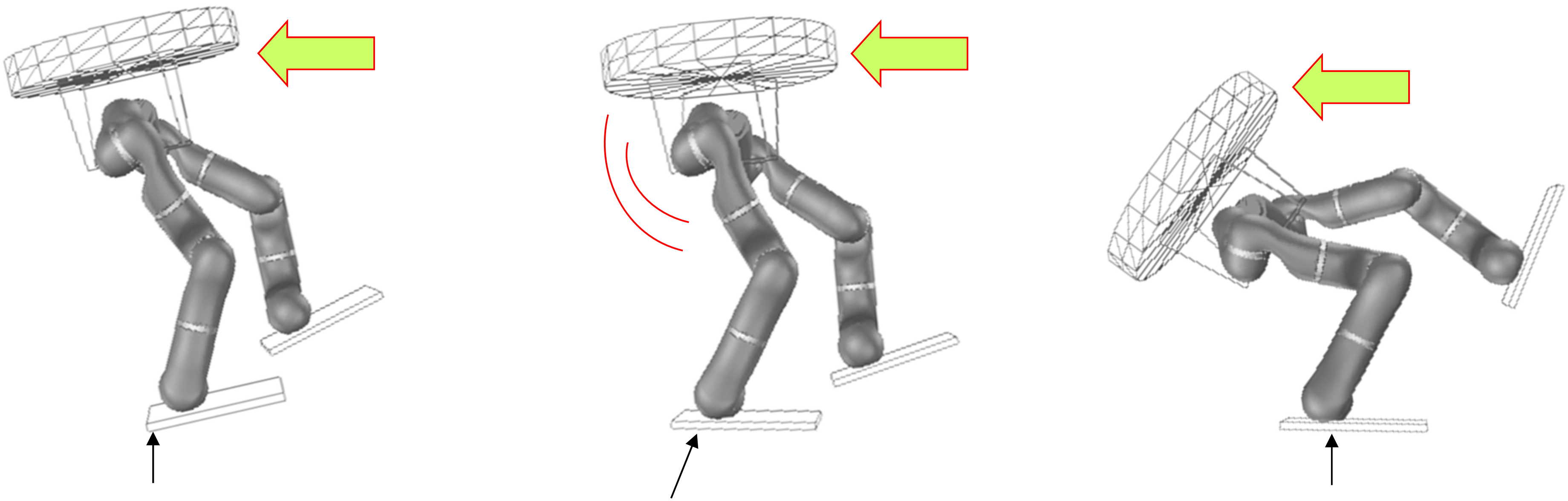
Part II: Balancing

1. Basics
2. ZMP based balancing (concentrated mass model)
3. Torque based balancing (multi-body model)

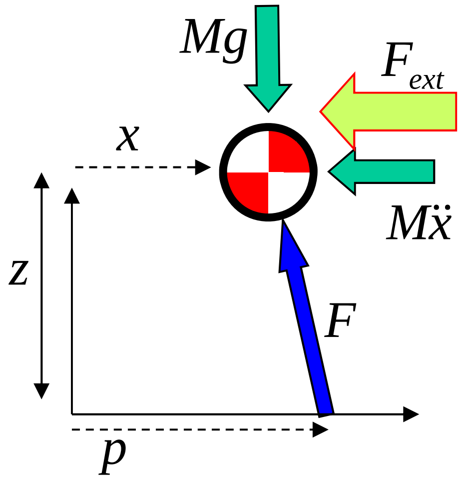
Part III: Walking Control

Motivation for compliant control

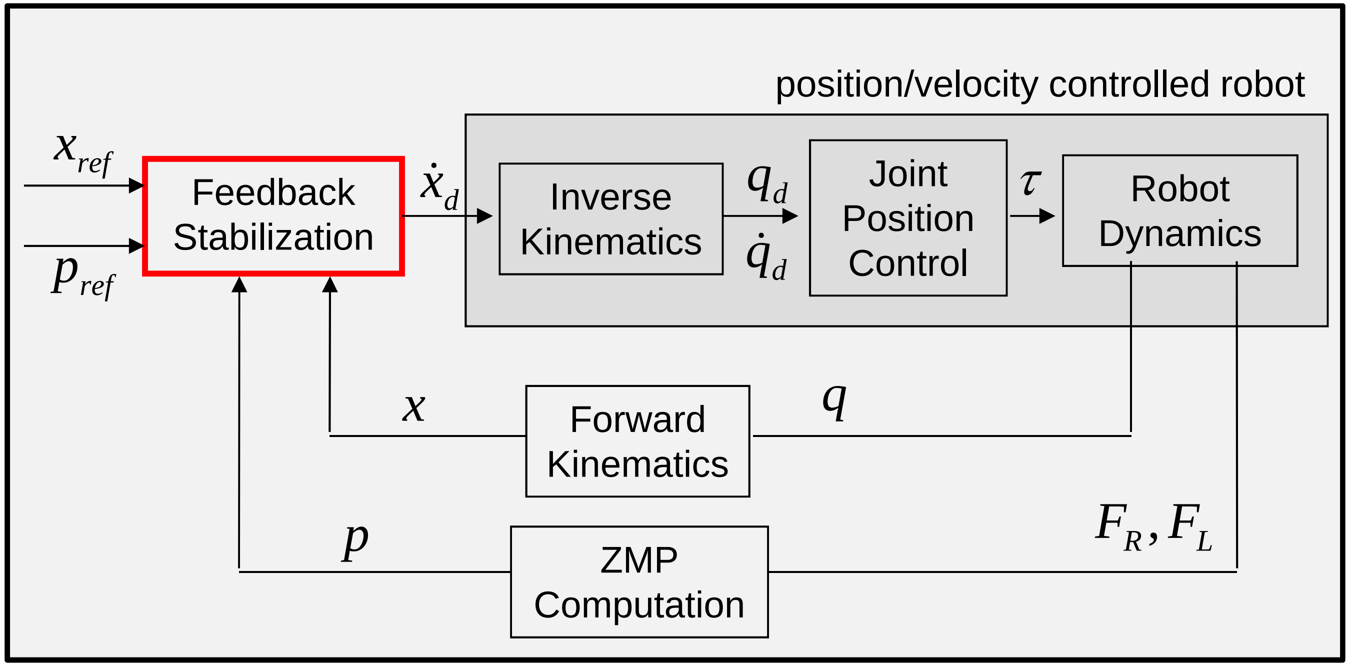
completely stiff | compliant control | fully compliant



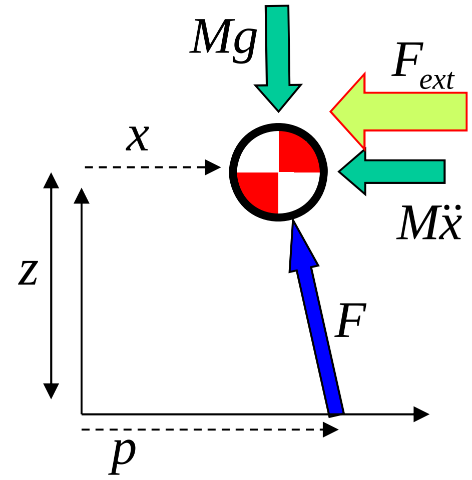
ZMP based balancing



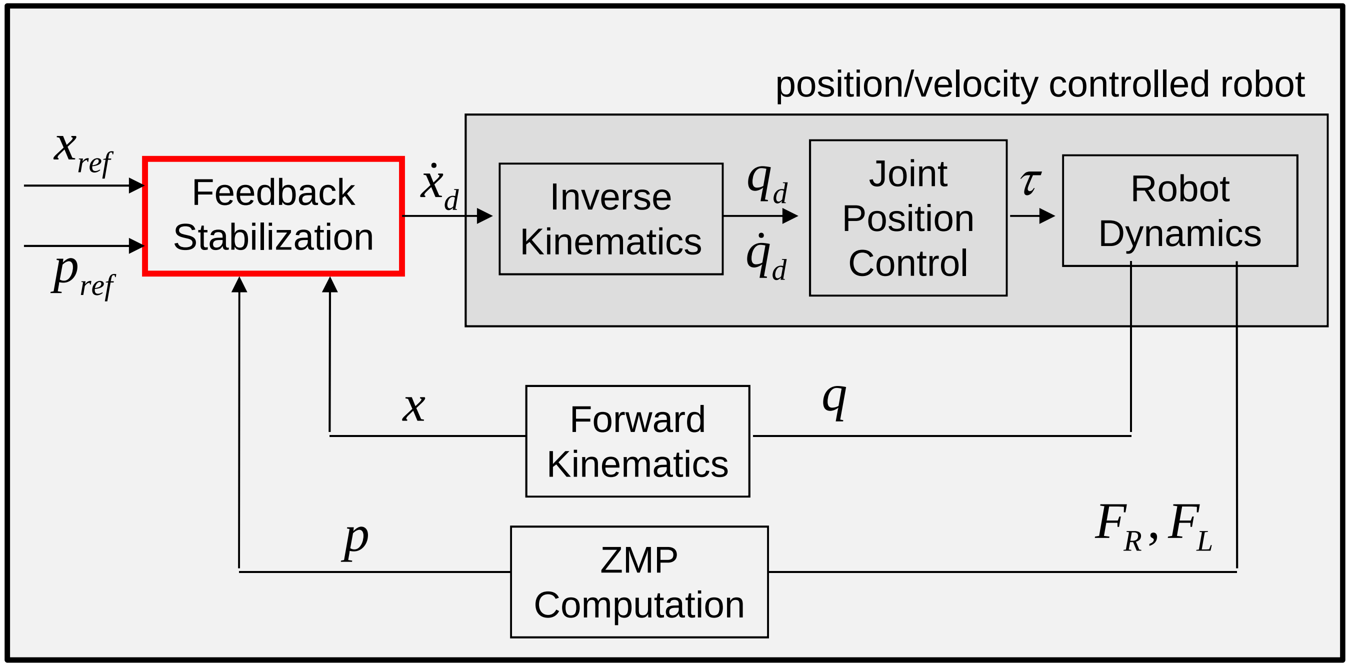
$$\frac{p-x}{z} = \frac{M\ddot{x}+F_{ext}}{Mg}$$



ZMP based balancing



$$\frac{p-x}{z} = \frac{M\ddot{x}+F_{ext}}{Mg}$$



Control law for stabilization:

$$\dot{x}_d = \dot{x}_{ref} - K_X(x_d - x_{ref}) + K_P(p - p_{ref})$$

Stability condition [*]: $K_X > \omega > K_P > 0$

Effective Stiffness:

$$K = \left(\frac{K_X}{K_P} - 1\right)\frac{Mg}{z}$$

Stability condition [*]: [Choi, Kim, Oh, and You, *Posture/Walking Control for Humanoid Robot Based on Kinematic Resolution of CoM Jacobian With Embedded Motion*, TRO, 2007].

ZMP Based balancing

Balancing + Vertical Motion

Balancing + Vertical Motion

+ Compliant Orientation

Overview

Part I: Modeling

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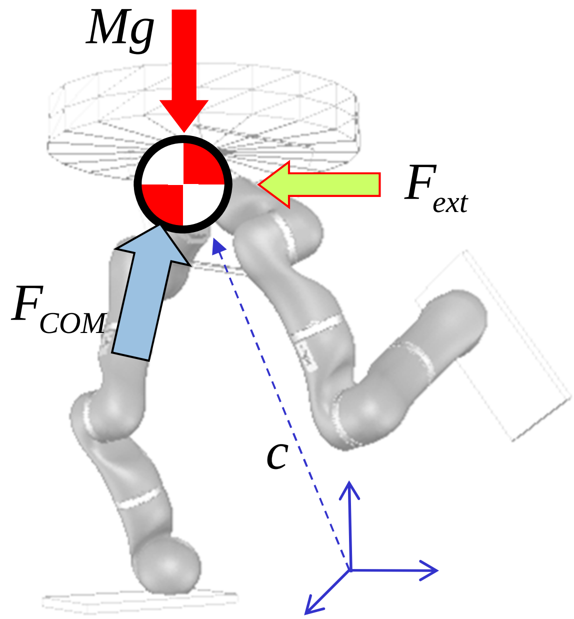
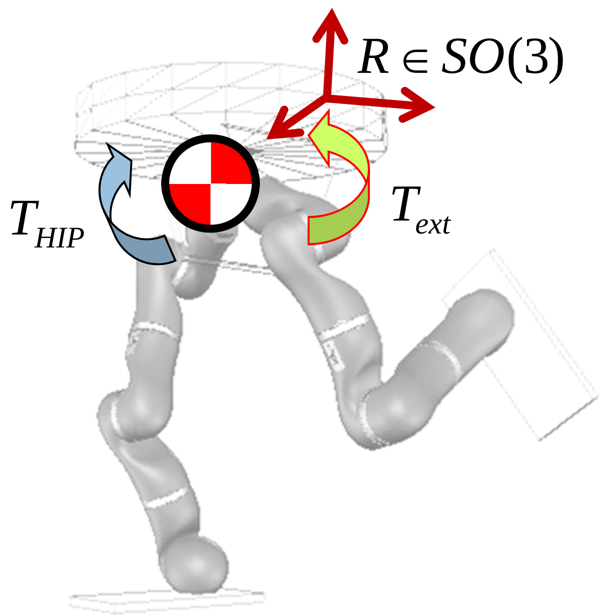
Balancing & Posture Control

Compliant COM control [Hyon & Cheng, 2006]

$$F_{COM} = Mg - K_P(c - c_d) - K_D(\dot{c} - \dot{c}_d)$$





Trunk orientation Control

$$T_{HIP} = \frac{\partial \dot{V}(R, K_R)}{\partial \omega} + D_R(\omega - \omega_d)$$

IMU measurements

Balancing & Posture Control

Compliant COM control [Hyon & Cheng, 2006]

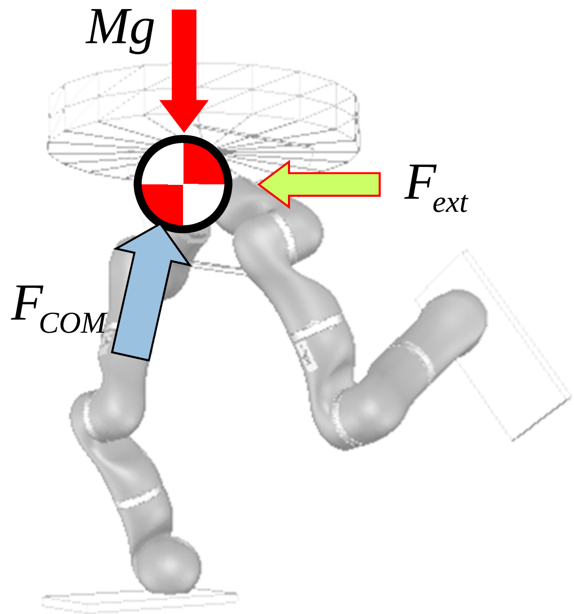
$$F_{COM} = Mg - K_P(c - c_d) - K_D(\dot{c} - \dot{c}_d)$$

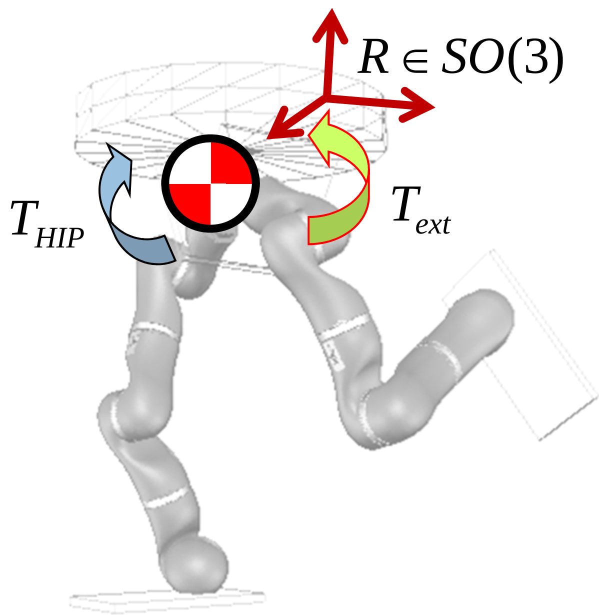
Desired wrench: $W_d = (F_{COM}, T_{HIP})$

Trunk orientation Control

$$T_{HIP} = \frac{\partial \dot{V}(R, K_R)}{\partial \omega} + D_R(\omega - \omega_d)$$

IMU measurements





Balancing & Posture Control

Compliant COM control [Hyon & Cheng, 2006]

$$F_{COM} = Mg - K_P(c - c_d) - K_D(\dot{c} - \dot{c}_d)$$

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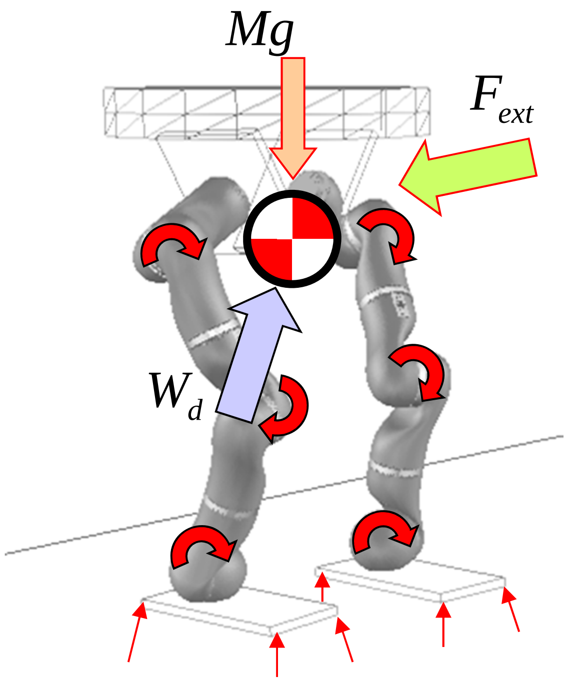
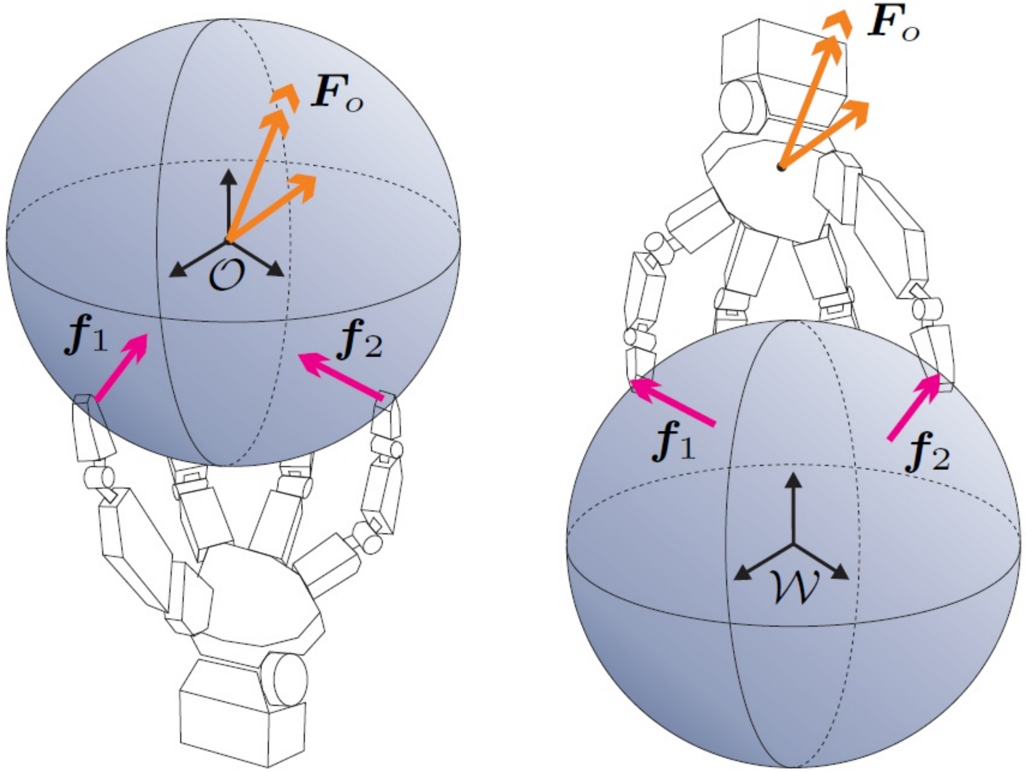
IMU measurements

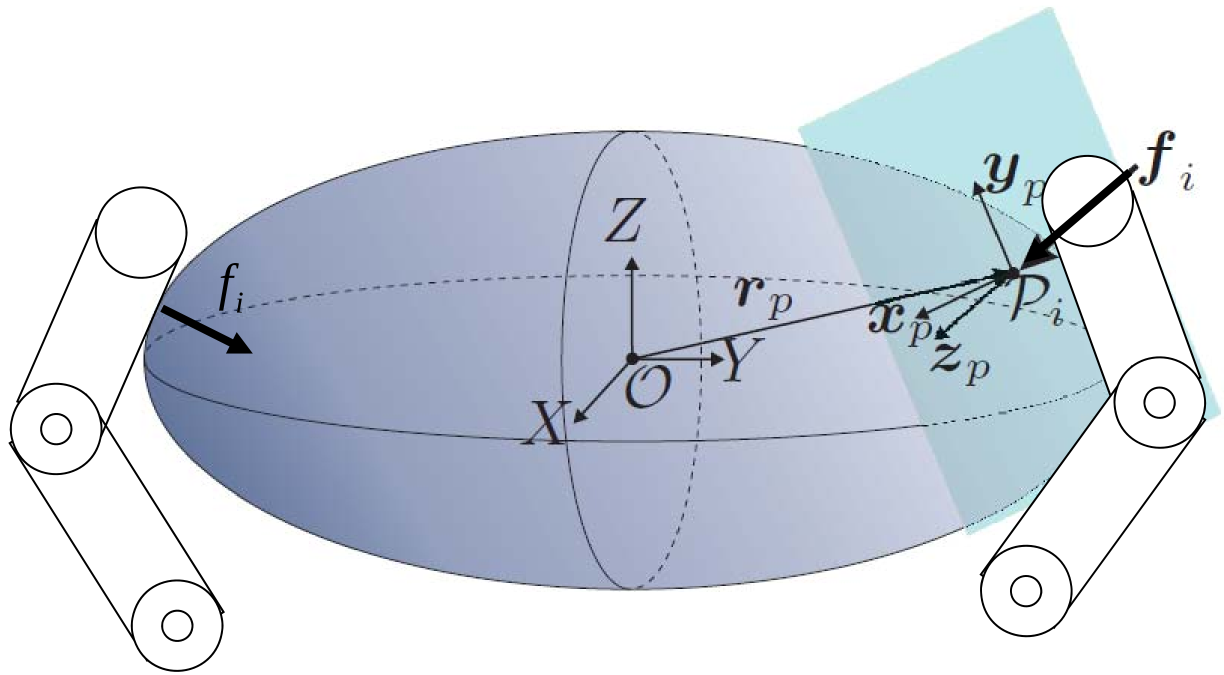


Grasping and Balancing

Force distribution: Similar problems!



Force Distribution in Grasping



Net wrench acting on the object:

$$\underbrace{W_O}_{se(3)} = G_1F_1 + \cdots + G_\eta F_\eta = \underbrace{\left[G_1 \cdots G_\eta\right]}_{\text{Grasp Map}} \underbrace{\begin{pmatrix} F_1 \\ \vdots \\ F_\eta \end{pmatrix}}_{F_C \in se(3)^\eta}$$

$$G_i = Ad^T_{PiO}$$

Well studied problem in grasping: Find contact wrenches $F_C \in FC^\eta$ such that a desired net wrench on the object is achieved.

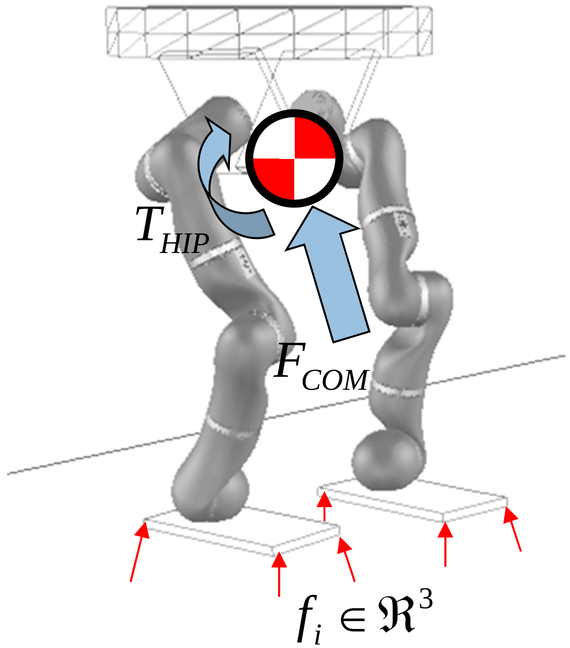
friction cone

Force distribution

Relation between balancing wrench & contact forces

$$G_i = \begin{bmatrix} R_i \\ \hat{p}_i R_i \end{bmatrix}$$

$$\boxed{W_d = \underbrace{\left[G_1 \cdots G_\eta\right]}_{\begin{bmatrix} G_F \\ G_T \end{bmatrix}} \underbrace{\begin{pmatrix} f_1 \\ \vdots \\ f_\eta \end{pmatrix}}_{f_C}}$$



Constraints:

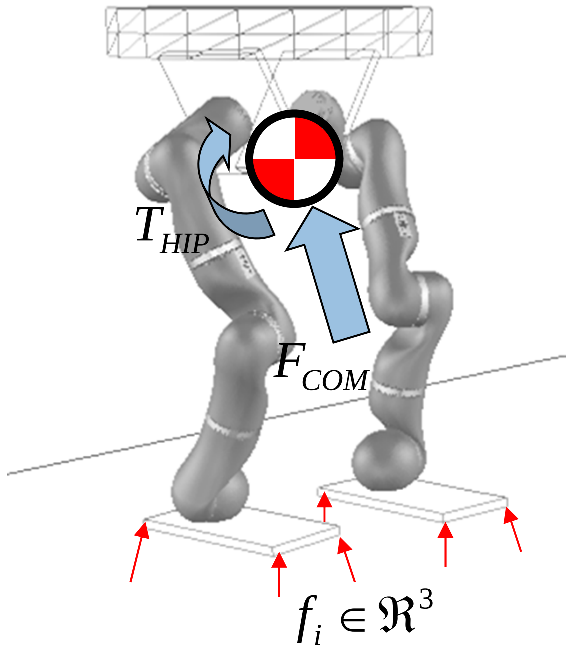
- Unilateral contact: $f_{i,z} > 0$ (implicit handling of ZMP constraints)
- Friction cone constraints

Force distribution

Relation between balancing wrench & contact forces

$$G_i = \begin{bmatrix} R_i \\ \hat{p}_i R_i \end{bmatrix}$$

$$\boxed{W_d = \underbrace{\left[G_1 \cdots G_\eta\right]}_{\begin{bmatrix} G_F \\ G_T \end{bmatrix}} \underbrace{\begin{pmatrix} f_1 \\ \vdots \\ f_\eta \end{pmatrix}}_{f_C}}$$



Constraints:

- Unilateral contact: $f_{i,z} > 0$ (implicit handling of ZMP constraints)
- Friction cone constraints

Formulation as a constraint optimization problem

$$f_C = \arg\min \left\{ \alpha_1 \left\| F_{COM} - G_F f_C \right\|^2 + \alpha_2 \left\| T_{HIP} - G_T f_C \right\|^2 + \alpha_3 \left\| f_C \right\|^2 \right\} \qquad \alpha_1 >> \alpha_2 >> \alpha_3$$

Contact force control via joint torques

Multibody robot model:

COM as a base coordinate → system structure with decoupled COM dynamics.

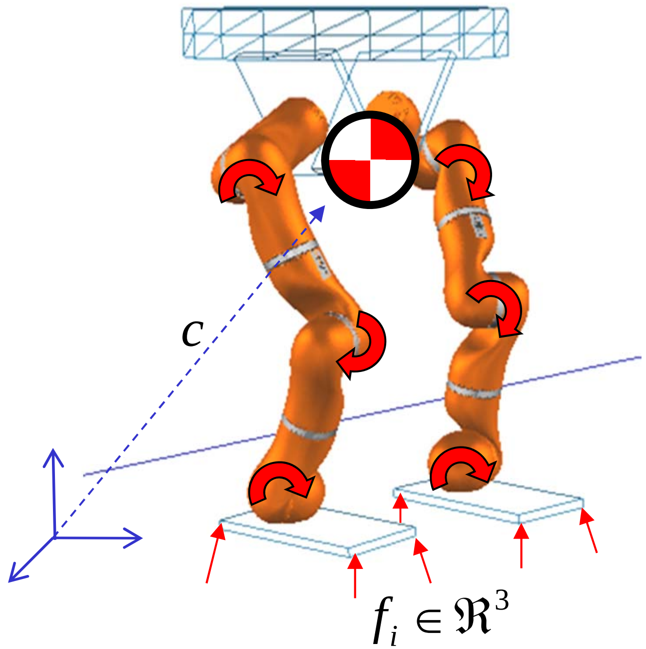
[Space Robotics], [Wieber 2005, Hyon et al. 2006]

$$\begin{bmatrix} M & 0 \\ 0 & \hat{M}(q) \end{bmatrix} \begin{pmatrix} \ddot{c} \\ \ddot{\hat{q}} \end{pmatrix} + \begin{bmatrix} 0 \\ \hat{C}(\hat{q}, \dot{\hat{q}}) \end{bmatrix} + \begin{bmatrix} -Mg \\ 0 \end{bmatrix} = \begin{pmatrix} 0 \\ u \end{pmatrix} - \sum_{i=r,l} \begin{bmatrix} I & 0 \\ J_i(\hat{q})^T \end{bmatrix} F_i$$

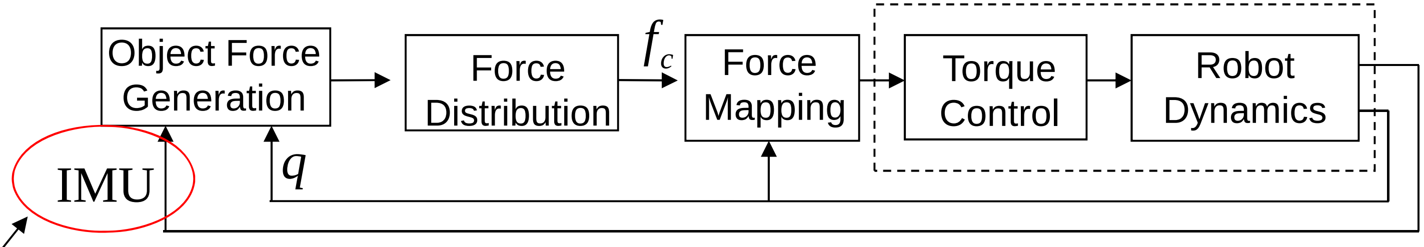
$$M\ddot{c} = Mg - \sum f_i$$

$$\boxed{\tau = \sum J_i(\hat{q})^T f_i}$$

Passivity based compliance control
(well suited for balancing)



Torque based balancing



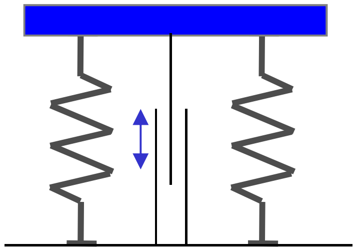
for orientation control and COM computation

Uncertain Foot Contact

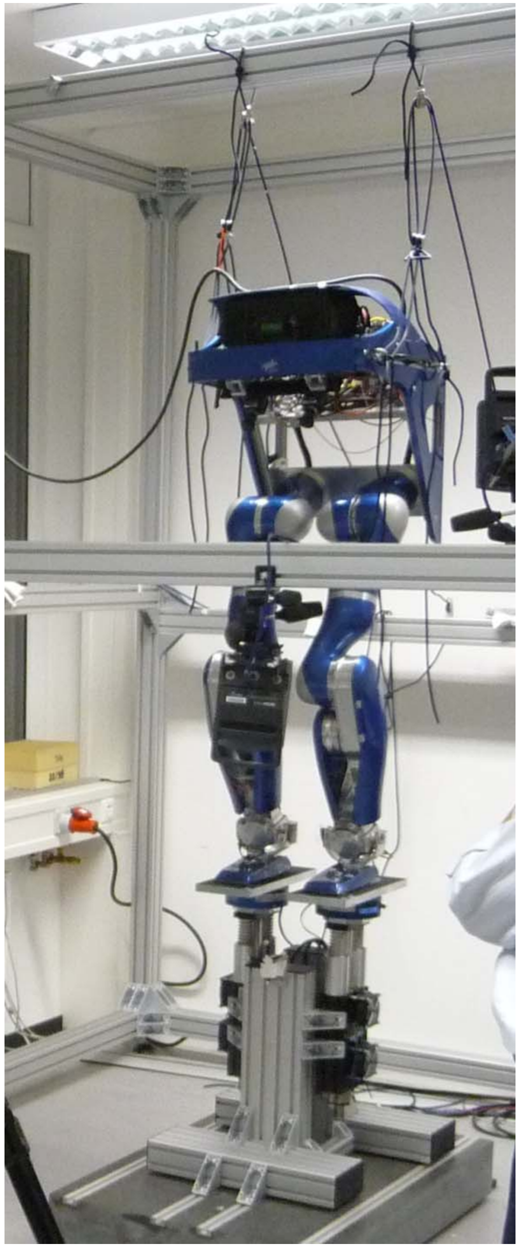
[Ott, Roa, Humanoids 2011, best paper award]

Experiments on a Perturbation Platform

- Leg perturbation setup
- Movable elastic platform

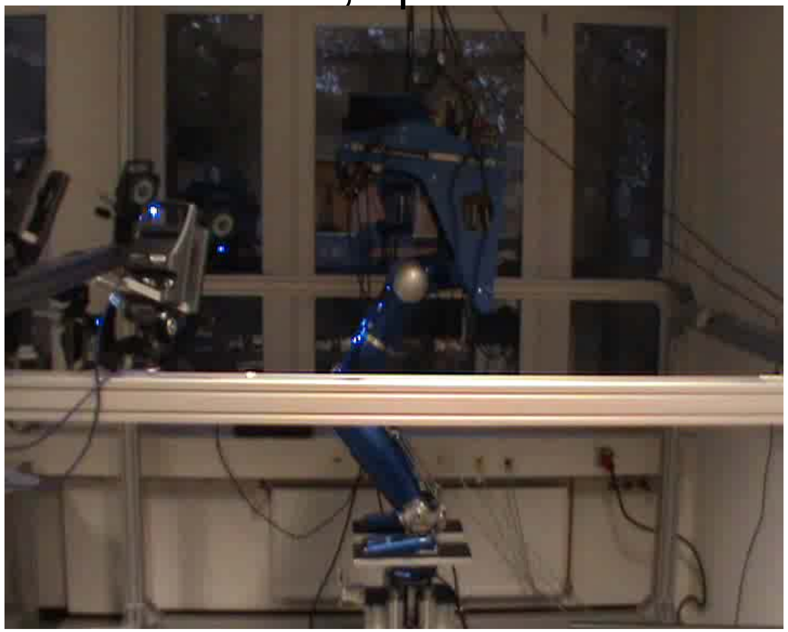


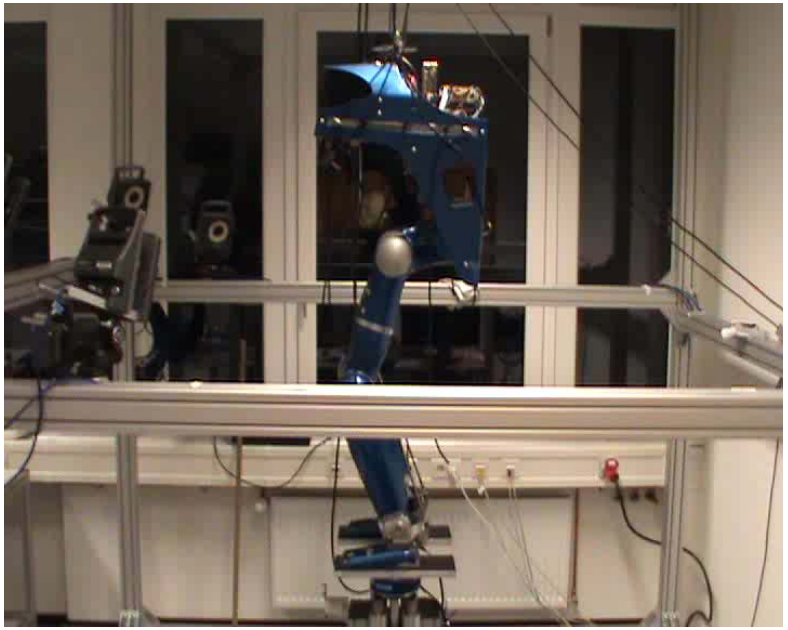
- Experimental evaluation of the robustness with respect to disturbances (frequency & amplitude) at the foot



Out of phase disturbance

synchronous disturbance
2mm, up to 8 Hz





Comparisons

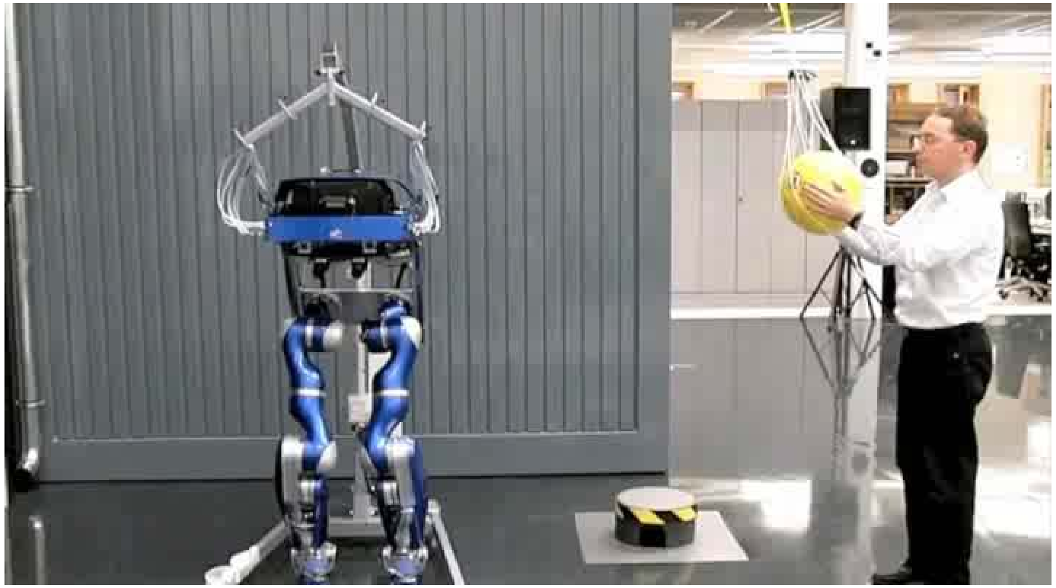
1) Impact experiments
2) Whole body interaction
3) Singularities

Comparison 1/3

1) Impact experiments

Position Based Control

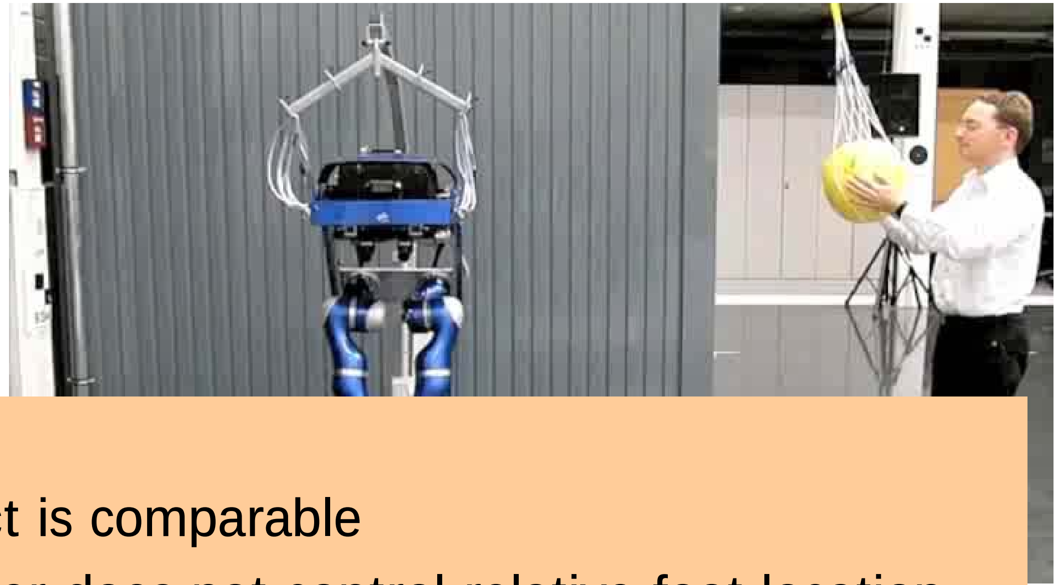
Torque Based Control



Comparison 1/3

1) Impact experiments

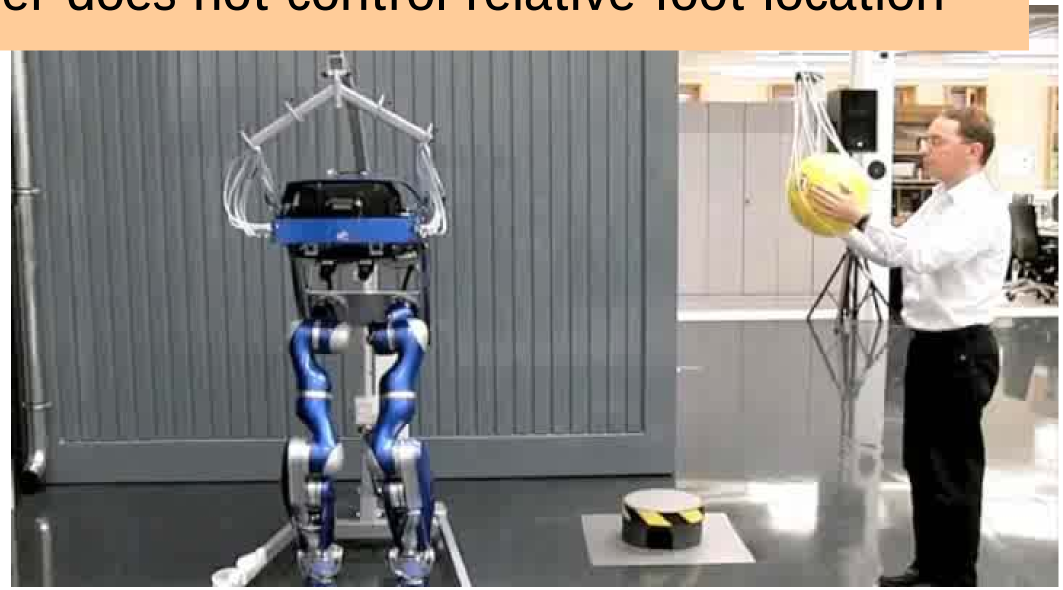
Position Based Control



Observations:

– Balancing after impact is comparable
– Torque based controller does not control relative foot location

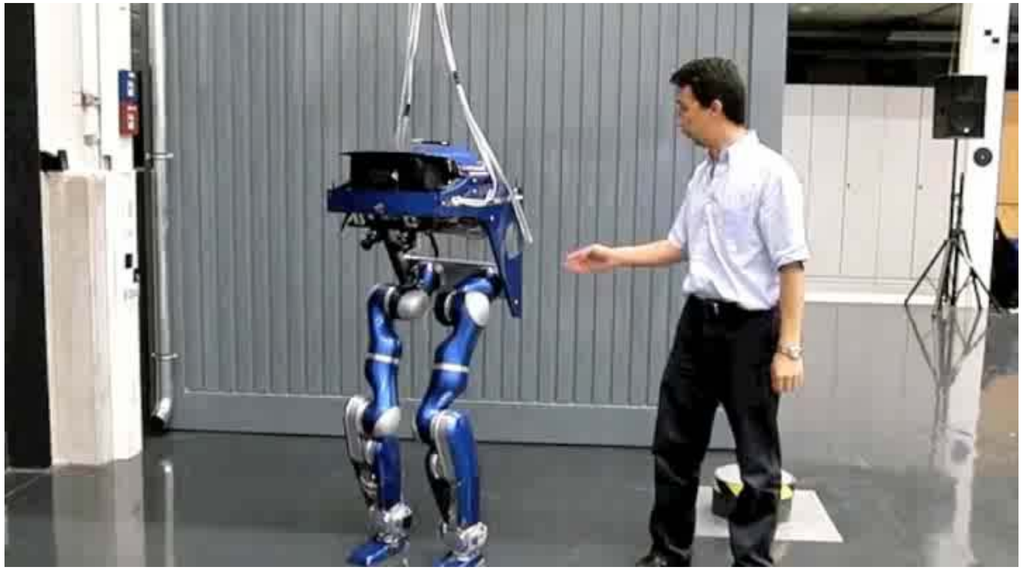
Torque Based Control



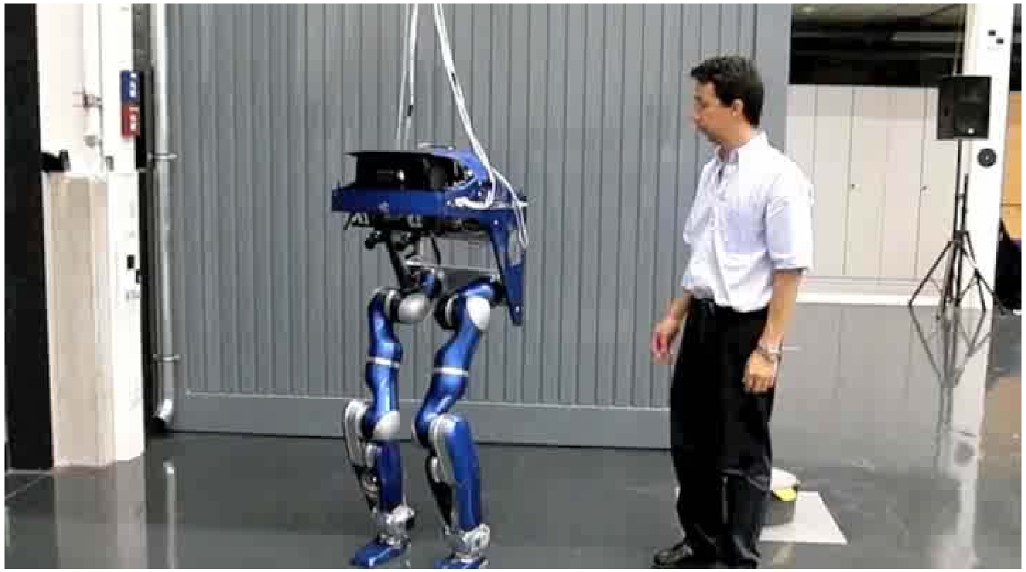
Comparison 2/3

3) Whole body interaction

Position Based Control



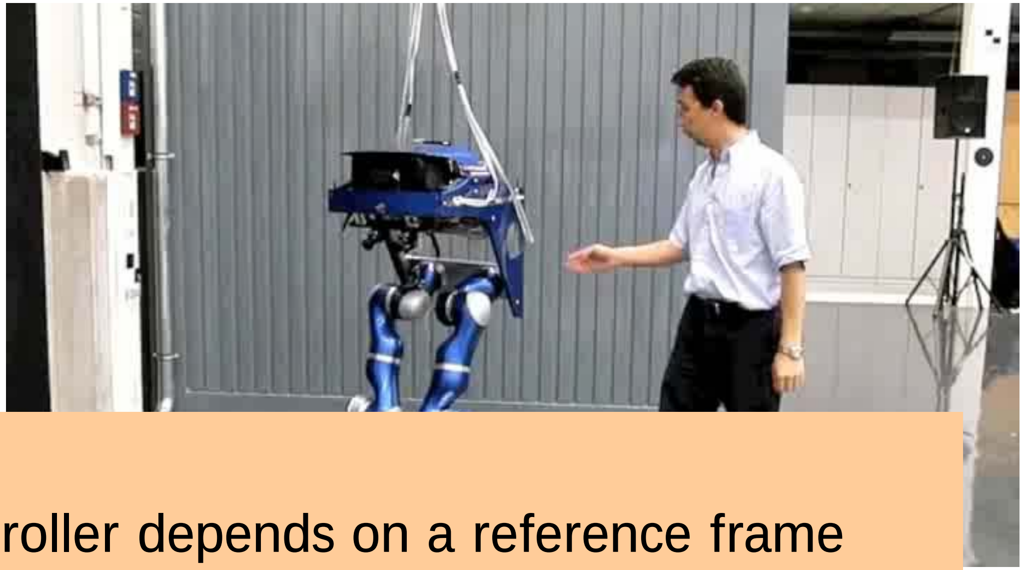
Torque Based Control



Comparison 2/3

3) Whole body interaction

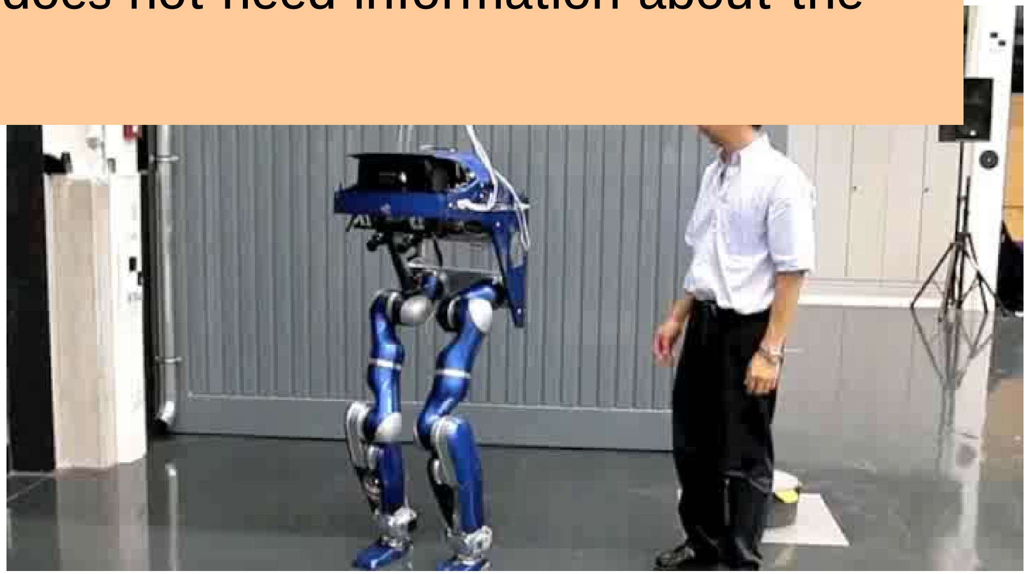
Position Based Control



Observations:

- Force sensor based controller depends on a reference frame
- Torque based controller does not need information about the point of contact

Torque Based Control



Comparison 3/3

4) Singular Configurations

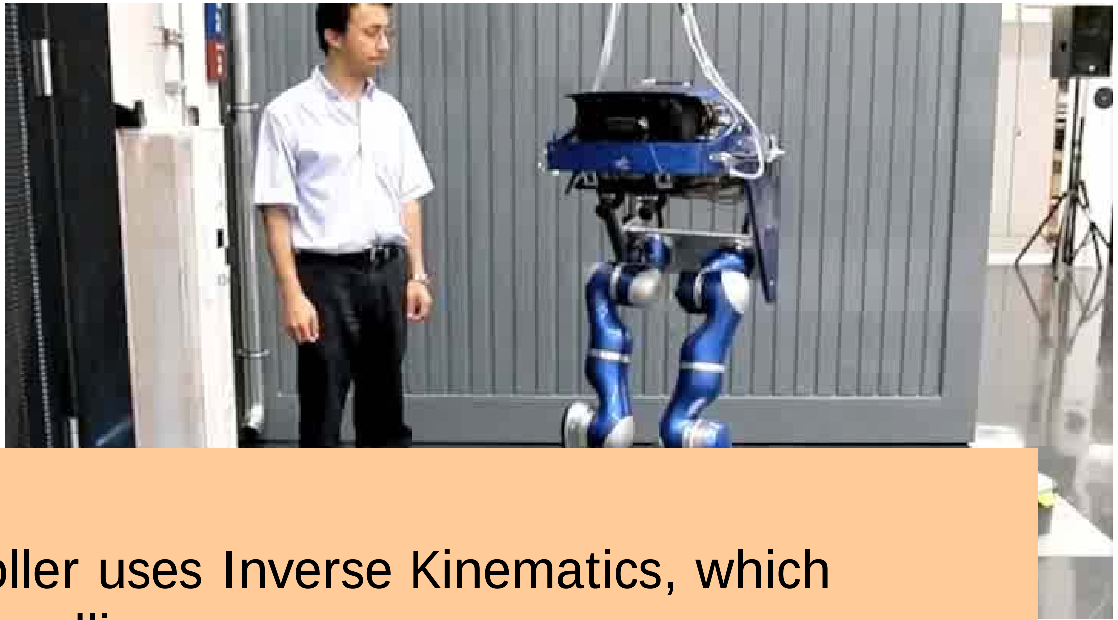
Position Based Control

Torque Based Control

Comparison 3/3

4) Singular Configurations

Position Based Control



Observations:

- Position based controller uses Inverse Kinematics, which requires singularity handling
- Torque based controller uses transposed Jacobian mapping, and thus is not affected by singularities

Torque Based Control

Balancing: Summary

- ✓ On flat floor both approaches allow for a compliant behavior
- ✓ Torque based controller shows independence on precise ground contact (force mapping based on IMU information)
- ✓ Admittance controller depends on a reference frame

Overview

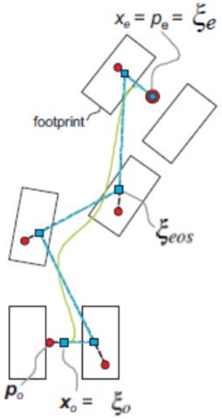
Part I: Modeling

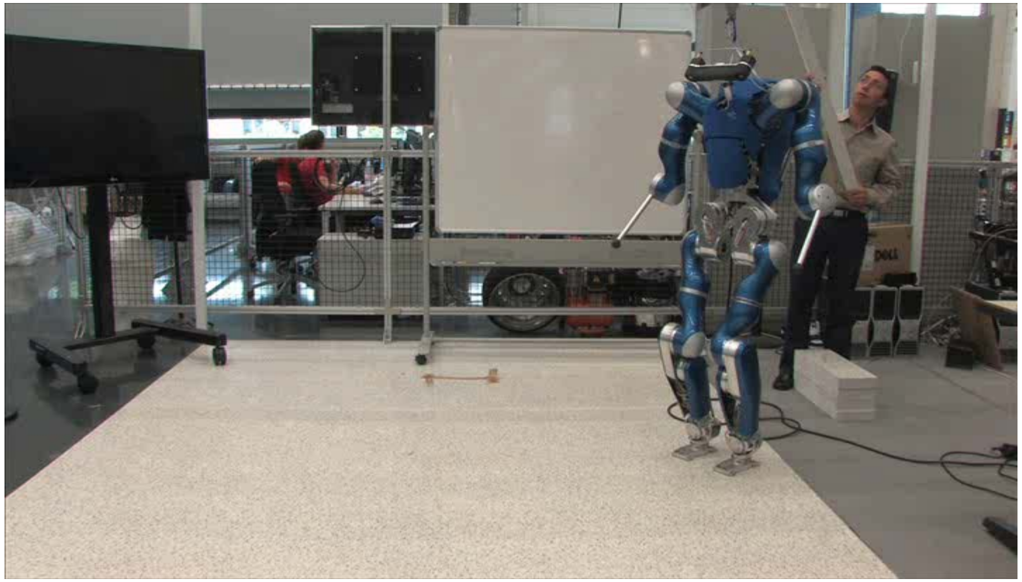
Part II: Balancing

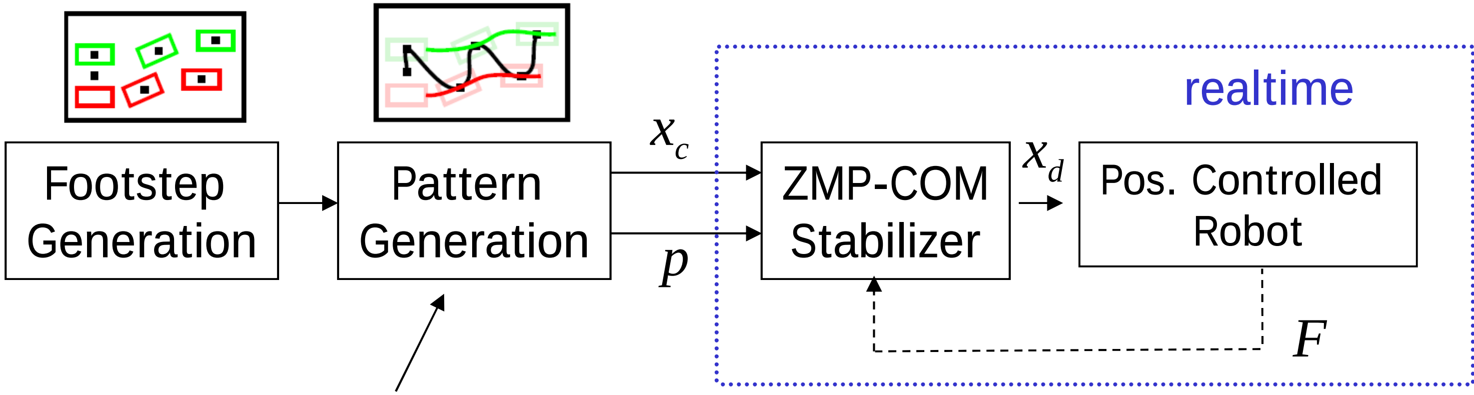
Part III: Walking Control

1. Walking pattern generation
2. Feedback control

Robot control based on conceptual models





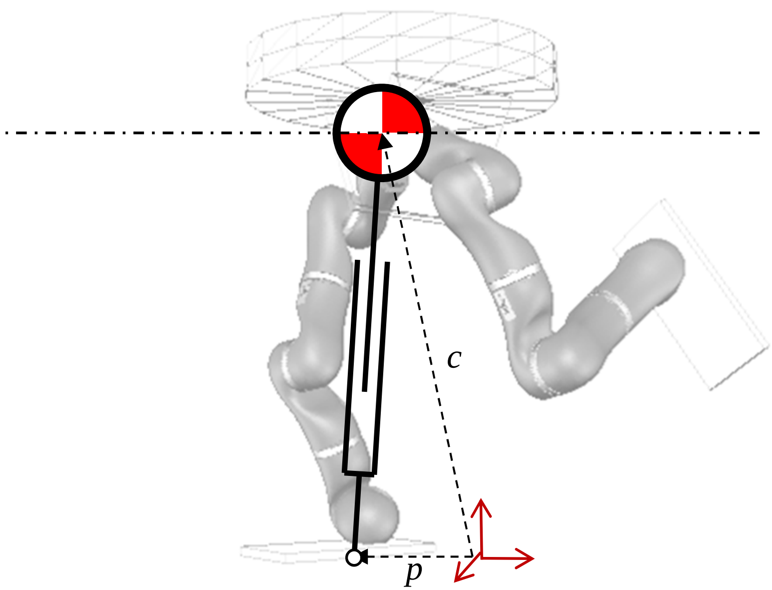


e.g. LQR Preview Control [Kajita, 2003]

Model Predictive Control [Wieber, 2006]

Mass concentrated model

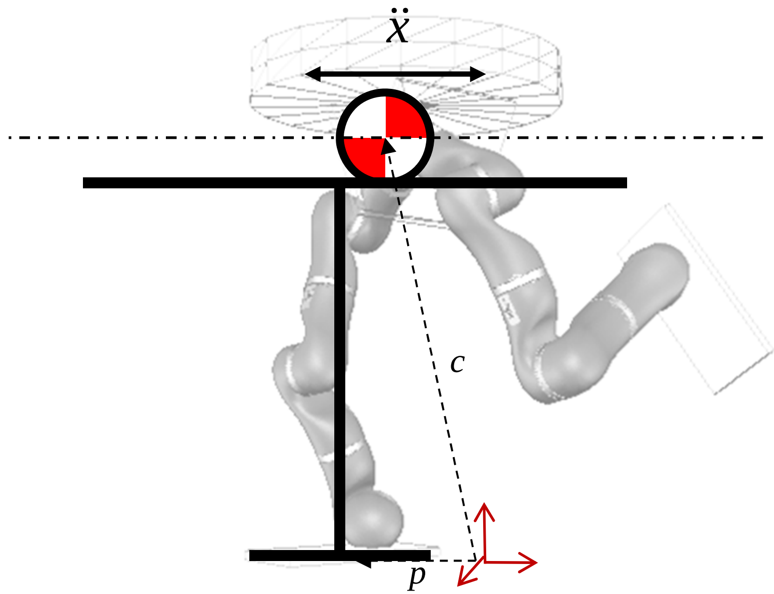
Linear Inverted Pendulum Model [Sugihara]



$$\ddot{x} = \frac{g}{z}(x - p)$$

$$\ddot{x} \leftarrow p$$

Cart-Table Model [Kajita]



$$p = x - \frac{z}{g}\ddot{x}$$

$$p \leftarrow \ddot{x}$$

Mass concentrated model

Continuous time control model

$$y = p$$

$$\bar{x} = \begin{bmatrix} x \\ \dot{x} \\ \ddot{x} \end{bmatrix}$$

$$u = \dddot{x}$$

$$\dot{\bar{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \bar{x} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

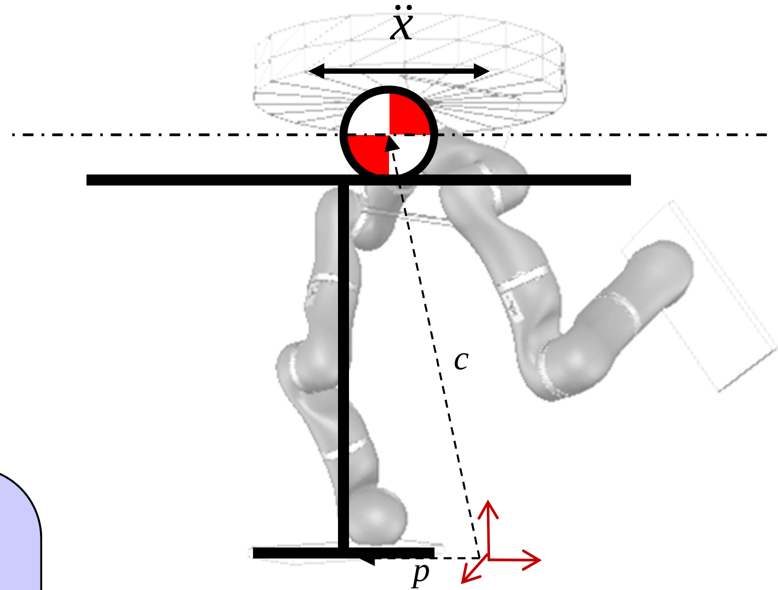
$$y = \begin{bmatrix} 1 & 0 & -\frac{z}{g} \end{bmatrix} \bar{x}$$

Discrete time model

$$\bar{x}(k+1) = \begin{bmatrix} 1 & T & T^2/2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix} \bar{x}(k) + \begin{bmatrix} T^3/6 \\ T^2/2 \\ T \end{bmatrix} u(k)$$

$$y(k) = \begin{bmatrix} 1 & 0 & -c_z/g \end{bmatrix} \bar{x}(k)$$

Cart-Table Model [Kajita]



$$p = x - \frac{z}{g}\ddot{x}$$

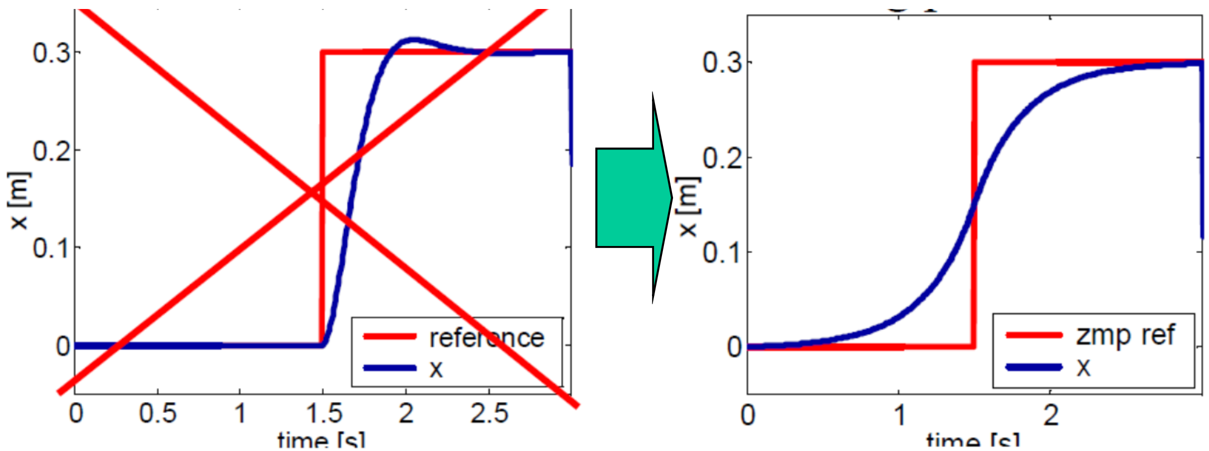
$$p \leftarrow \ddot{x}$$

LQR Preview Control

On a winding road, we steer a car by watching ahead,
by previewing the future reference.

- Concept and naming
  [Sheridan 1966]
- LQ optimal controller
  [Tomizuka and Rosenthal 1979]
  [Katayama et.al 1985]

How to use future information about the reference?



0.3
0.2
0.1
0
x [m]
0
0.5
1
1.5
2
2.5
time [s]
reference
x
0.3
0.2
0.1
0
x [m]
0
1
2
time [s]
zmp ref
x

LQR Preview Control

Time-discrete system:

$$x(k+1) = Ax(k) + Bu(k) \qquad x(k) \in \Re^n$$

$$y(k) = Cx(k) \qquad y(k) \in \Re^p$$

Assumptions:

Assume *(A,B)* is stabilizable & *(C,A)* is detectable, and [**] $rank\begin{bmatrix} 0 & C \\ B & A-I \end{bmatrix} = p+n$

[**] Ensures that the system has no transmission zero at z=1.

Reference output: Assume known for N future time steps $y_{ref}(k)$

Cost function:

$$J = \sum_{k=0}^{\infty} e(k)^T Q_e e(k) + \Delta x(k)^T Q_x \Delta x(k) + \Delta u(k)^T R \Delta u(k)$$

$$e(k) = y(k) - y_{ref}(k)$$

$$\Delta x(k) = x(k) - x(k-1)$$

$$\Delta u(k) = u(k) - u(k-1)$$

Q_e, R positive definite.

- Uses differential control input → integral action.
- Uses the output error compared to reference signal.

LQR Preview Control

Time-discrete system:

$$x(k+1) = Ax(k) + Bu(k)$$
$$y(k) = Cx(k)$$

$$x(k) \rightarrow \Delta x(k)$$
$$u(k) \rightarrow \Delta u(k)$$

Modified system representation:

$$\begin{bmatrix} e(k+1) \\ \Delta x(k+1) \end{bmatrix} = \begin{bmatrix} I & CA \\ 0 & A \end{bmatrix} \begin{bmatrix} e(k) \\ \Delta x(k) \end{bmatrix} + \begin{bmatrix} CB \\ B \end{bmatrix} \Delta u(k) + \begin{bmatrix} -I \\ 0 \end{bmatrix} \Delta y_{ref}(k+1)$$

[**] Ensures that this system is stabilizable.

LQR Preview Control

Time-discrete system:

$$x(k+1) = Ax(k) + Bu(k)$$

$$y(k) = Cx(k)$$

$$x(k) \rightarrow \Delta x(k)$$

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[**] Ensures that this system is stabilizable.

How to handle future reference input?

$$x_d(k) = \left[\Delta y_{ref}(k+1), \cdots, \Delta y_{ref}(k+N) \right]$$

System augmentation
(for next N reference input values)

Dynamics of the new state:

$$x_d(k+1) = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & \ddots & \ddots & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}}_{A_d} x_d(k)$$

LQR Preview Control

Modified system representation:

$$\underbrace{\begin{bmatrix} e(k+1) \\ \Delta x(k+1) \\ x_d(k+1) \end{bmatrix}}_{z(k+1)} = \begin{bmatrix} I & CA & \begin{bmatrix} -I & 0 \end{bmatrix} \\ 0 & A & 0 \\ 0 & 0 & A_d \end{bmatrix} \underbrace{\begin{bmatrix} e(k) \\ \Delta x(k) \\ x_d(k) \end{bmatrix}}_{z(k)} + \begin{bmatrix} CB \\ B \\ 0 \end{bmatrix} \Delta u(k)$$

Cost function:

$$J = \sum_{k=0}^{\infty} e(k)^T Q_e e(k) + \Delta x(k)^T Q_x \Delta x(k) + \Delta u(k)^T R \Delta u(k)$$

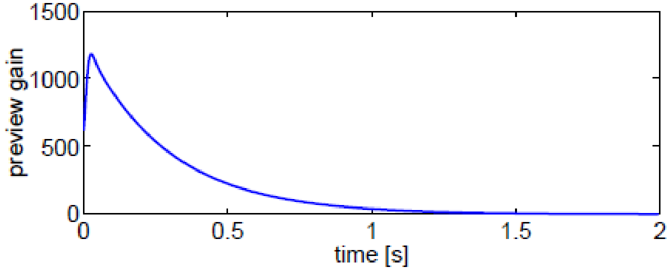
$$J = \sum_{k=0}^{\infty} z(k)^T \begin{bmatrix} Q_e & 0 & 0 \\ 0 & Q_x & 0 \\ 0 & 0 & 0 \end{bmatrix} z(k) + \Delta u(k)^T R \Delta u(k)$$

Standard LQR Design for augmented system!

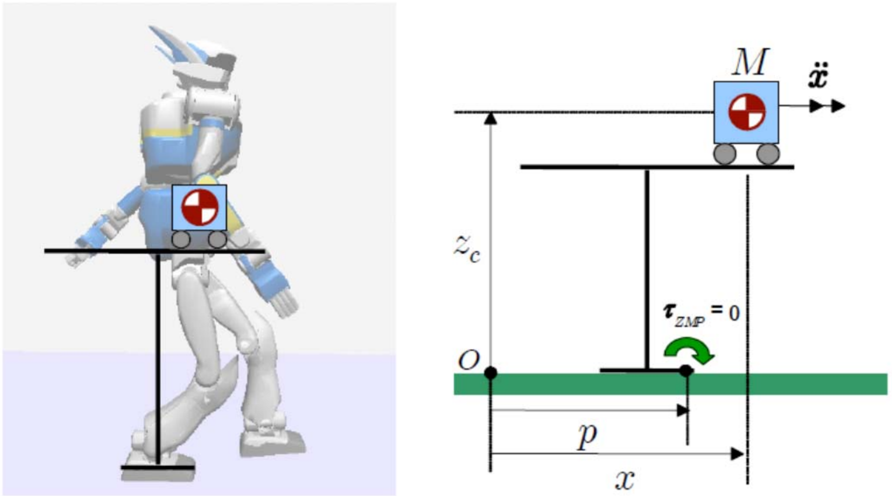
Preview Control

Control law: $$\Delta u(k) = Kz(k) = \begin{bmatrix} K_e & K_\Delta & K_d \end{bmatrix} \begin{bmatrix} e(k) \\ \Delta x(k) \\ x_d(k) \end{bmatrix}$$

$$u(k) = K_e \sum_{i=0}^{k} \left(y(i) - y_{ref}(i) \right) + K_\Delta x(k) + \sum_{i=1}^{N} K_d(i) y_{ref}(k+i)$$



Example Application: Walking Pattern Generation



Simplified Model: Cart Table Model

(Kajita 2003)

p ... ZMP (Zero Moment Point)
loosely speaking: Point on the sole where the reduced contact force is acting.

x ... Position of the CoM

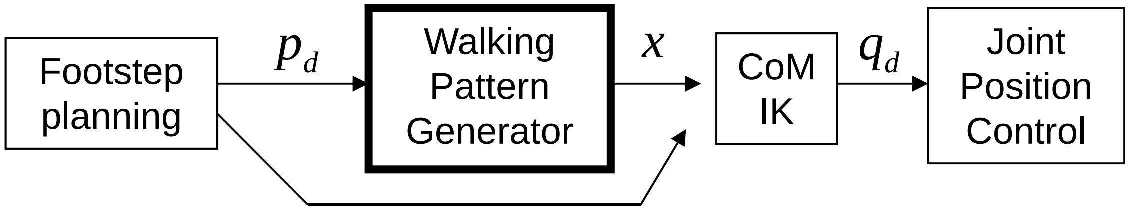
$$\bar{x} = \begin{bmatrix} x \\ \dot{x} \\ \ddot{x} \end{bmatrix}$$

$$u = \dddot{x}$$
$$y = p$$

$$\bar{x}(k+1) = \begin{bmatrix} 1 & T & T^2/2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix} \bar{x}(k) + \begin{bmatrix} T^3/6 \\ T^2/2 \\ T \end{bmatrix} u(k)$$

$$y(k) = \begin{bmatrix} 1 & 0 & -z/g \end{bmatrix} \bar{x}(k)$$

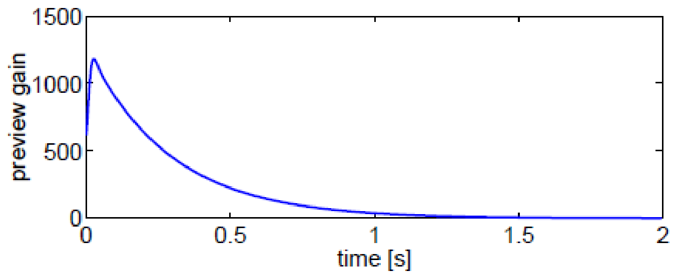
Example Application: Walking Pattern Generation



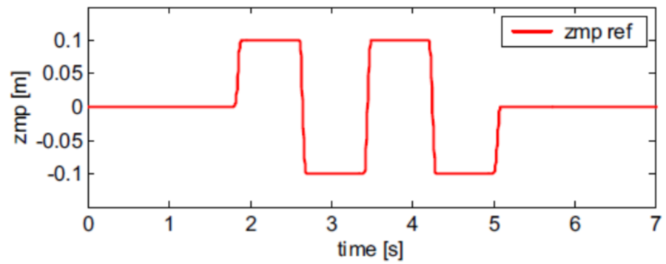
Preview Control:

T = 5 ms
N = 400

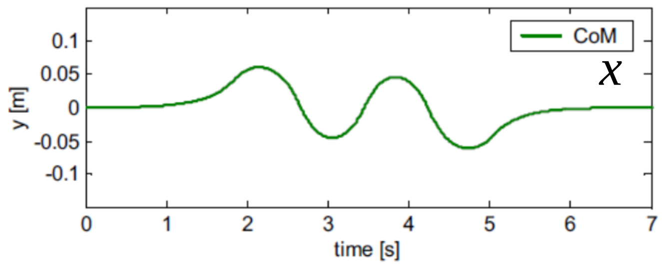
Q_x = zeros(3,3);
Q_e = 1.0;
R = 1e-6;



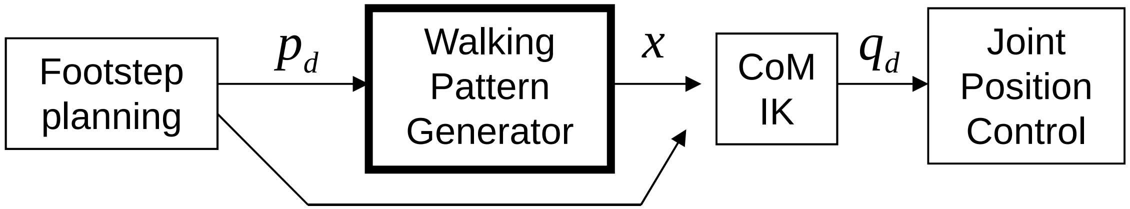
Target ZMP pattern



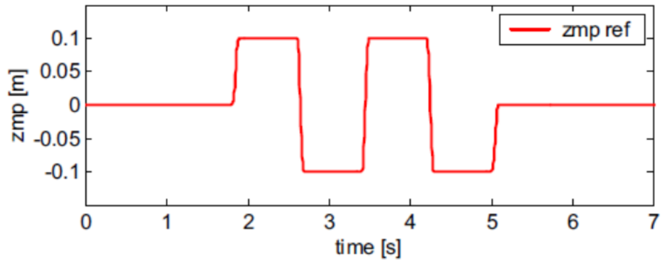
Trajectory of center of mass



Example Application: Walking Pattern Generation



Target ZMP pattern



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Q_x = zeros(3,3);
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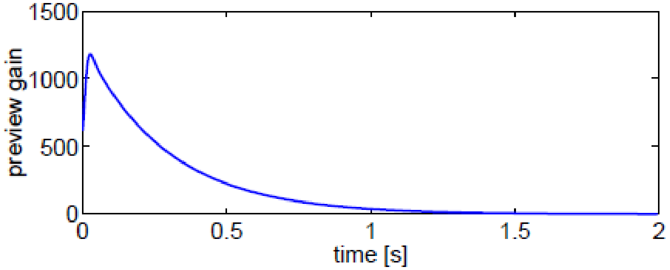
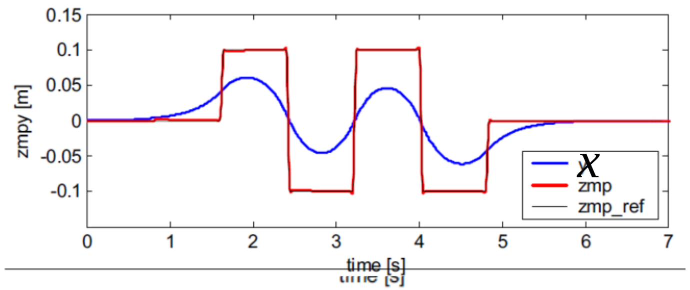




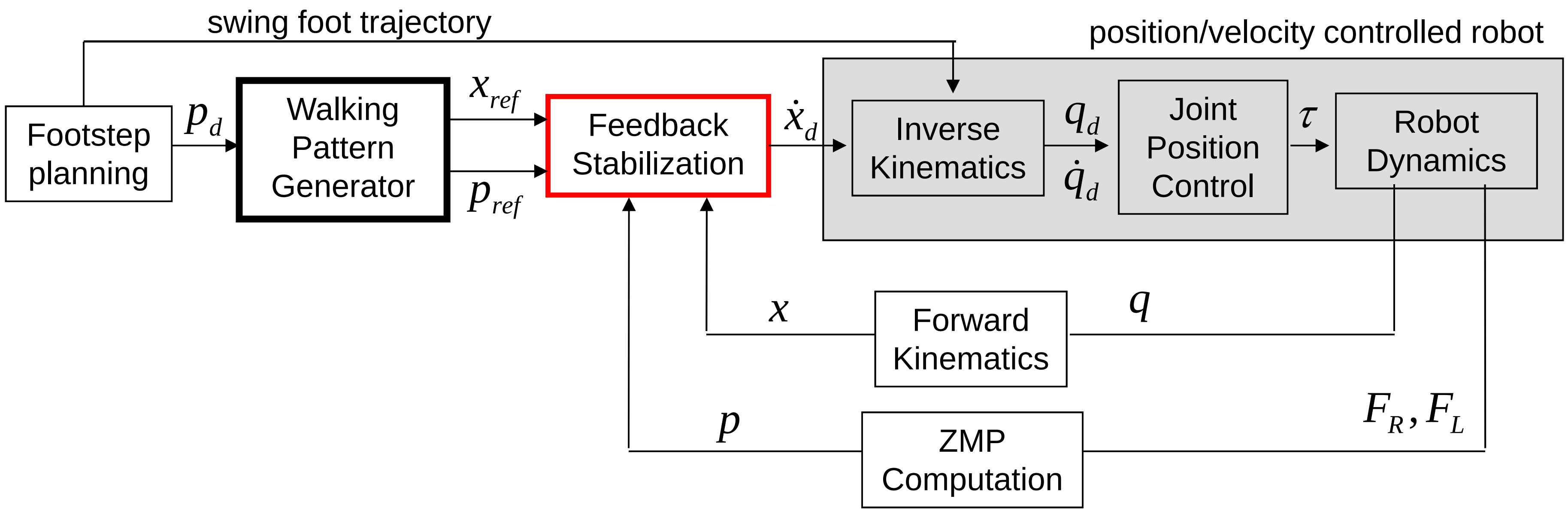
Properties of Preview Control

- Efficient implementation,
  controller design can be computed offline
- Allows to incorporate predictive information
- ZMP contstraints are not considered explicitely
- Trajectory based approach

Extensions

  – Model predictive control (handle zmp constraints explicitely, optimization over a finite control horizon)
  – Trajectory generation → feedback control
  – Dynamic filter

Feedback Stabilization



Control law for stabilization:

$$\dot{x}_d = \dot{x}_{ref} - K_X(x_d - x_{ref}) + K_P(p - p_{ref})$$

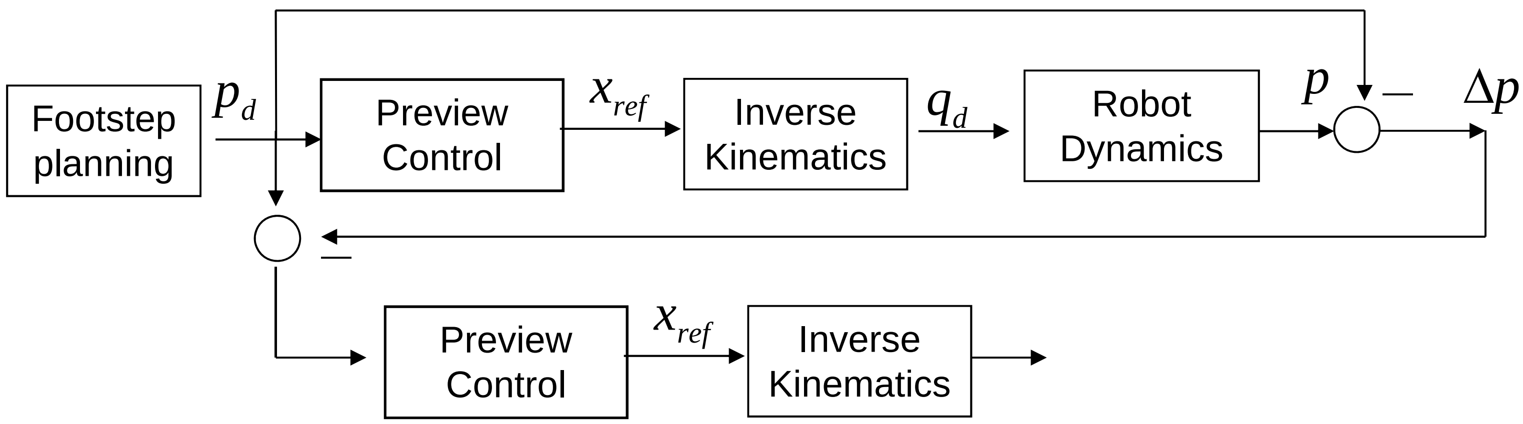
Stability condition [*]: $K_X > \omega > K_P > 0$

Stability condition [*]: [Choi, Kim, Oh, and You, *Posture/Walking Control for Humanoid Robot Based on Kinematic Resolution of CoM Jacobian With Embedded Motion*, TRO, 2007].

Dynamic filter

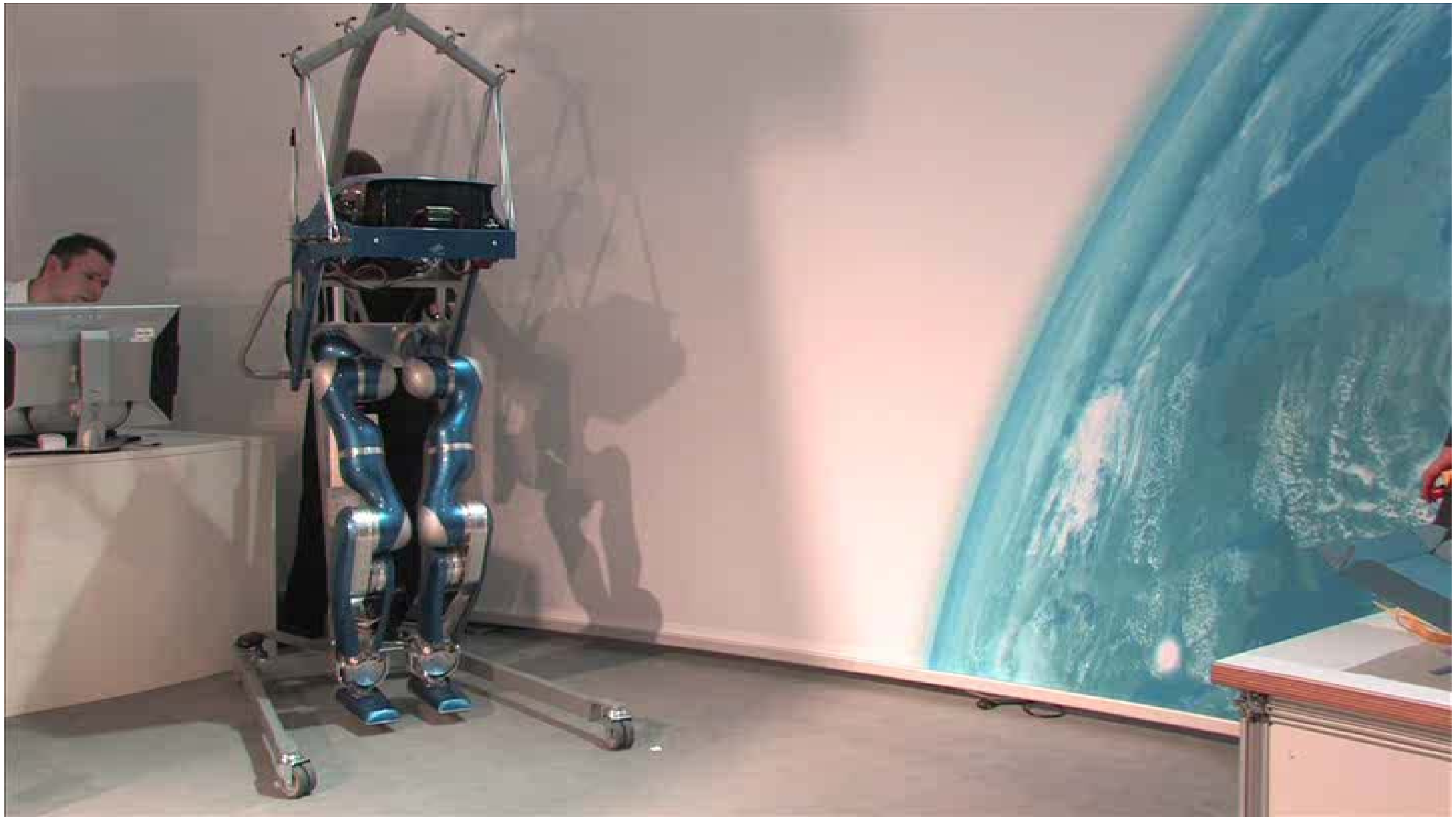
Correction of the error due to model simplification

Requires computation of the multi-body dynamics



DLR-Biped

ZMP basierte Gangregelung



Präsentiert auf der Industriemesse Automatica, Juni 2010

Overview

Part I: Modeling

Part II: Balancing

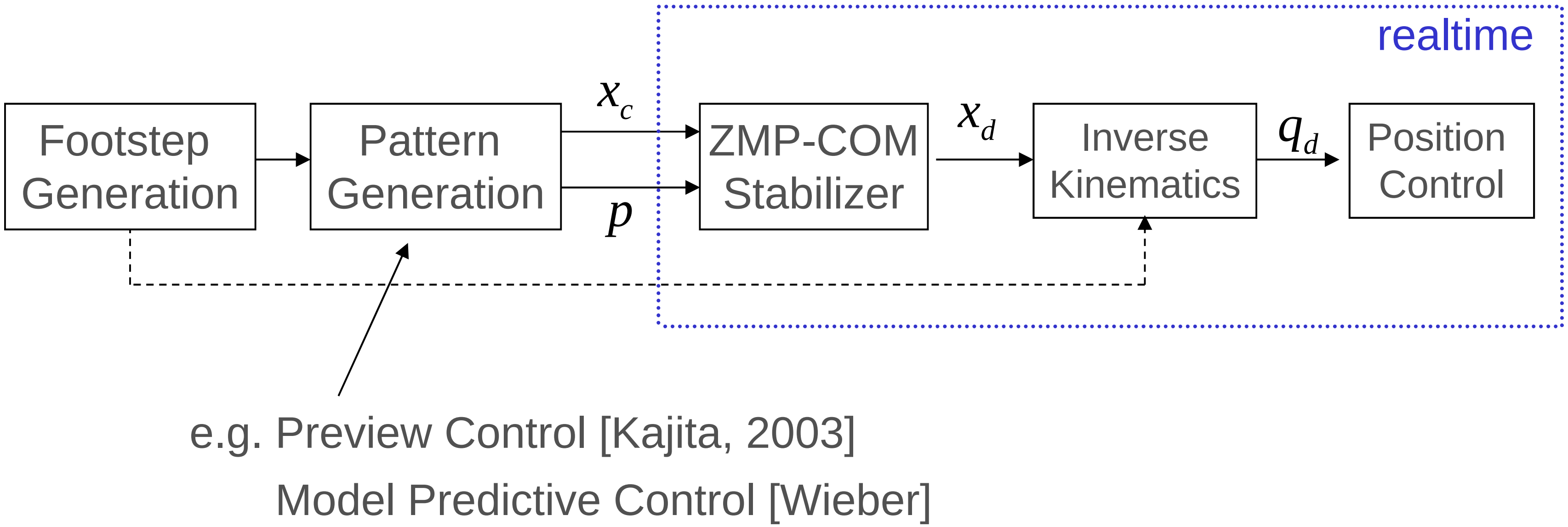
Part III: Walking Control

1. Walking pattern generation
2. Feedback control

Walking Control

State of the art walking control for fully actuated robots

– Pattern Generator for desired CoM and ZMP motion
– ZMP based Stabilizer



HELMHOLTZ ASSOCIATION

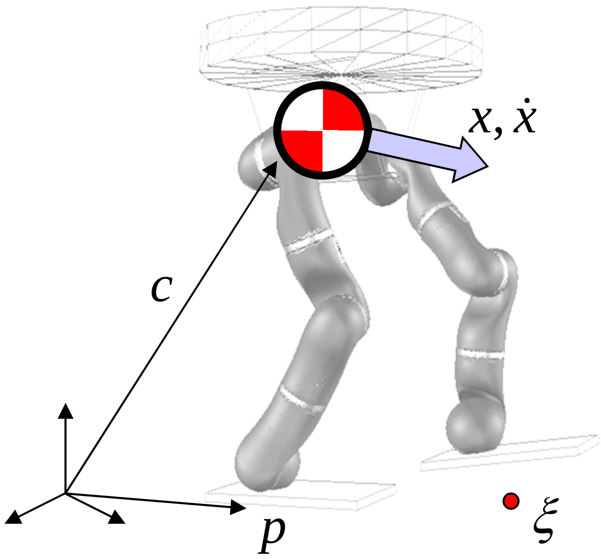
Walking Control used at DLR

[Englsberger, capture point control]

Walking Stabilization

- Core concept:
  Capture point control
- Generalization (3D)
  Stairs, etc …

- Predictive Control (MPC)
- Reactive step adaptation



(Pratt 2006, Hof 2008)

$$\ddot{x} = \omega^2 (x - p)$$

$$\xi = x + \frac{\dot{x}}{\omega}$$

$$(x, \dot{x}) \downarrow (x, \xi)$$

$$\begin{pmatrix} \dot{x} \\ \dot{\xi} \end{pmatrix} = \begin{bmatrix} -\omega & \omega \\ 0 & \omega \end{bmatrix} \begin{pmatrix} x \\ \xi \end{pmatrix} + \begin{bmatrix} 0 \\ -\omega \end{bmatrix} p$$

$p \to$ [capture point] $\xrightarrow{\xi}$ [COM] $\xrightarrow{x}$

capture point: open loop unstable

COM: exp. stable

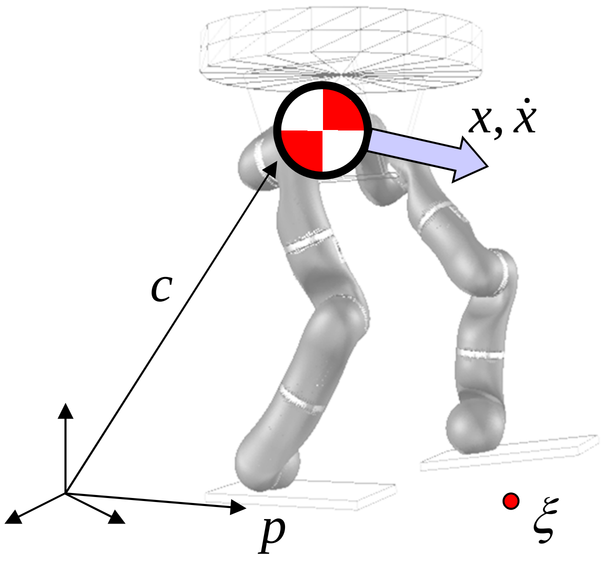
Walking Control used at DLR

[Englsberger, capture point control]

Walking Stabilization

- Core concept:
  Capture point control
- Generalization (3D)
  Stairs, etc …

- Predictive Control (MPC)
- Reactive step adaptation



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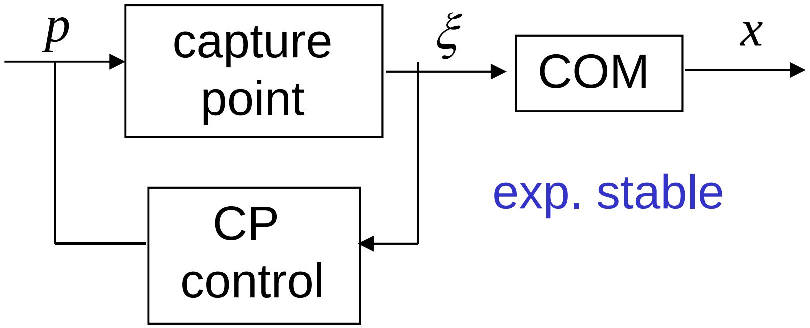
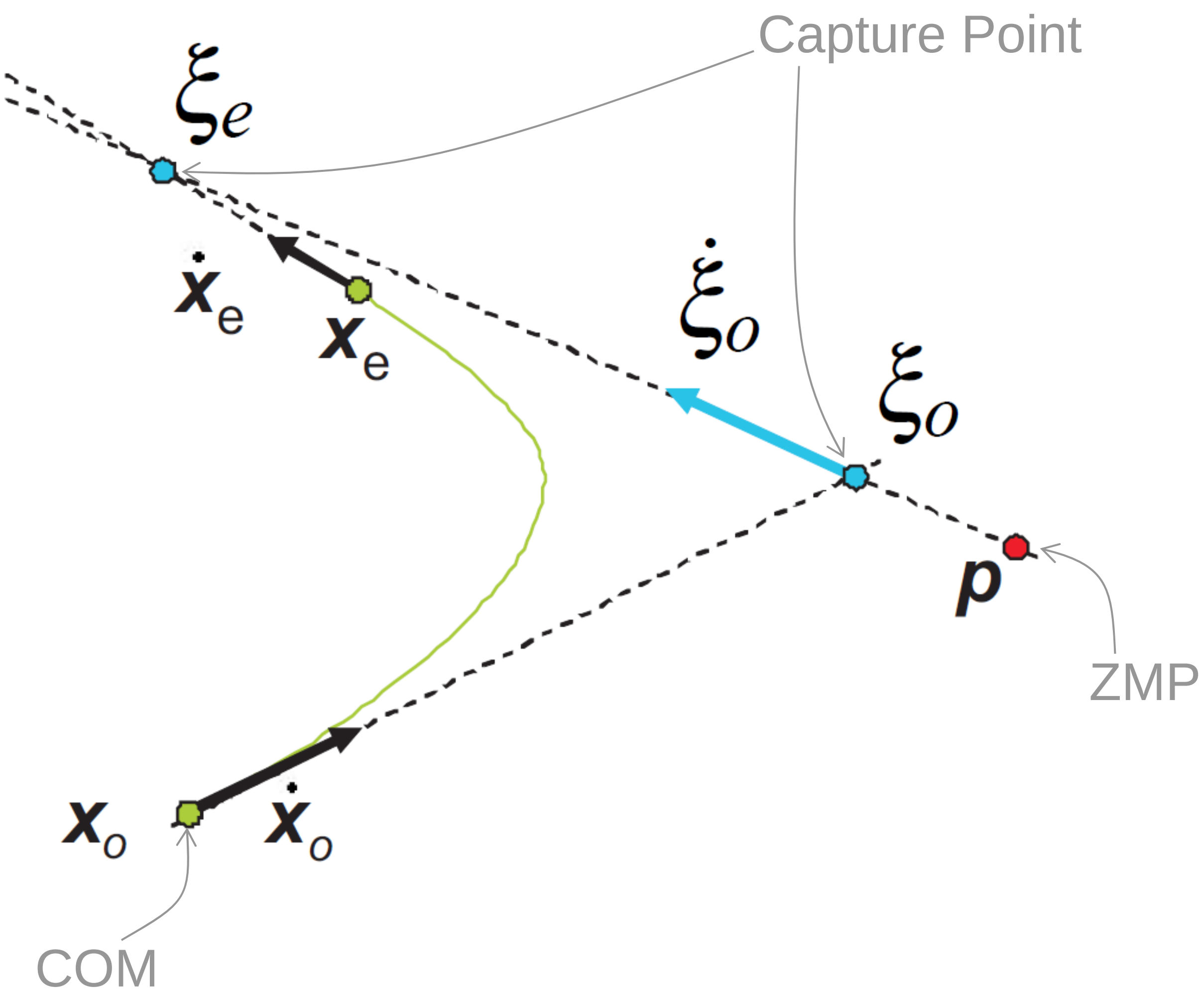


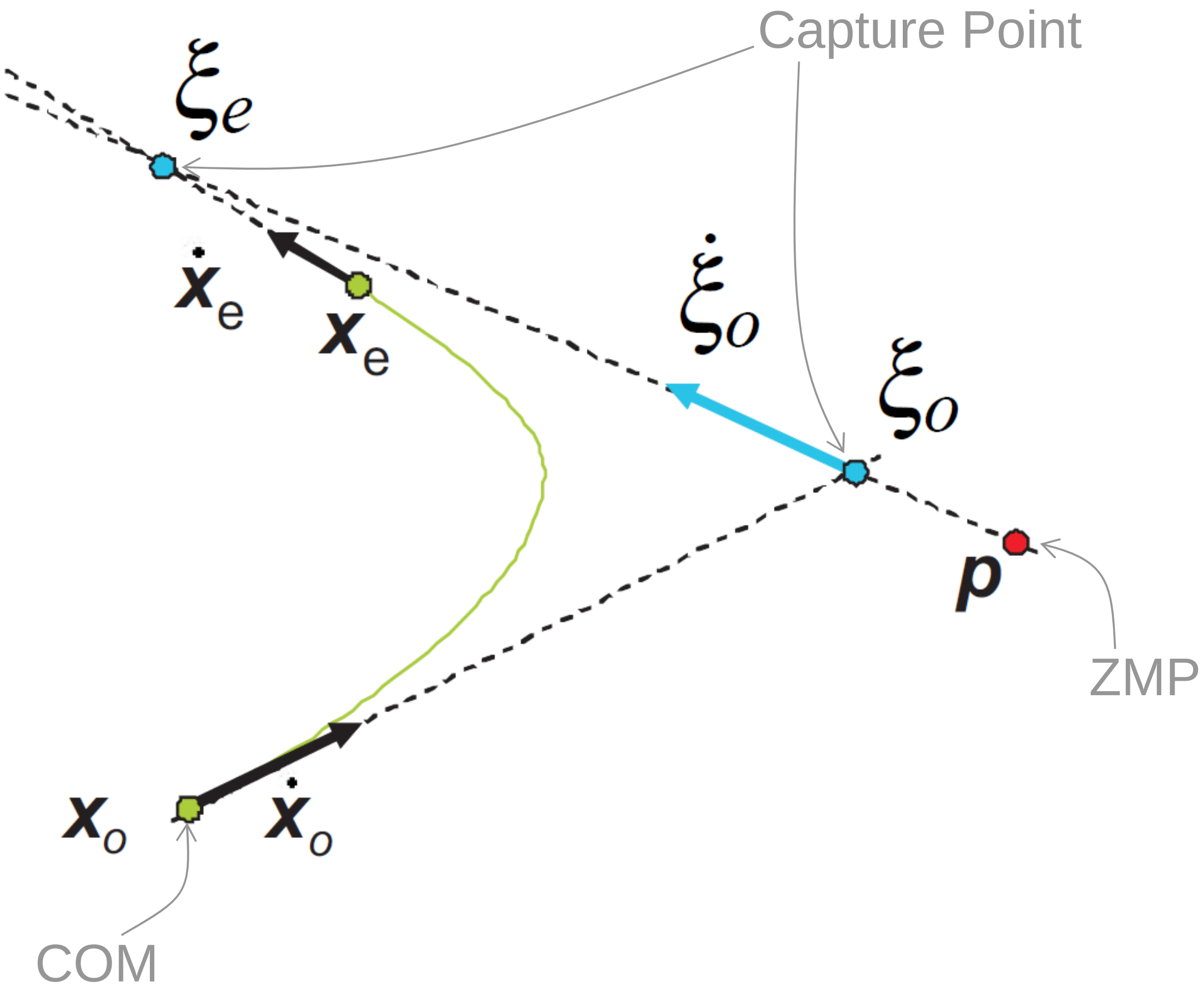
exp. stable

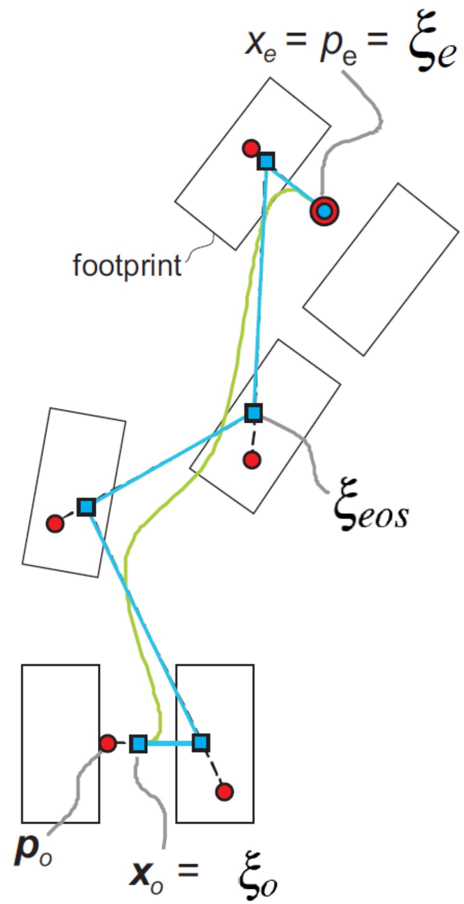
Capture Point Dynamik



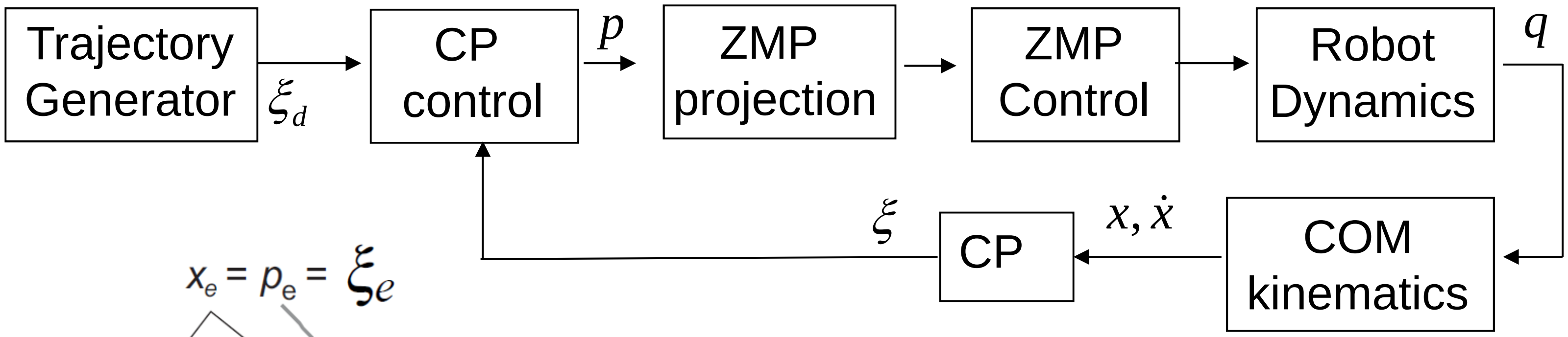
- COM velocity always points towards CP
- ZMP „pushes away“ the CP on a line
- COM follows CP

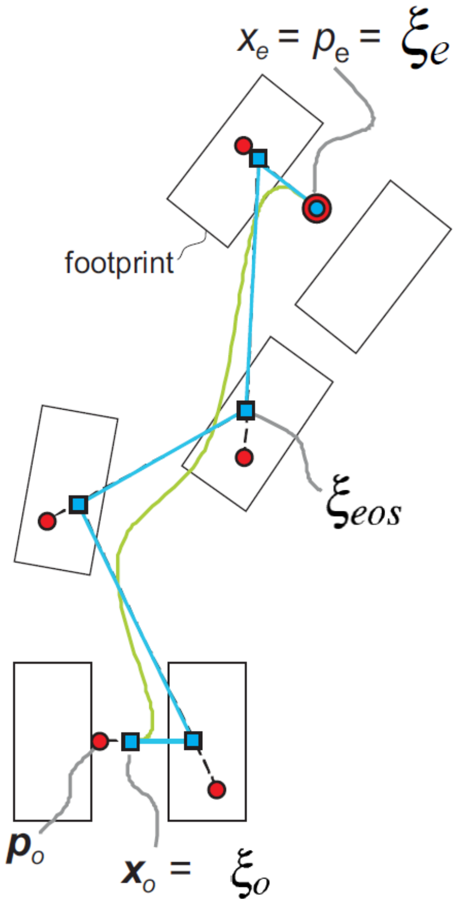
Capture Point Dynamik

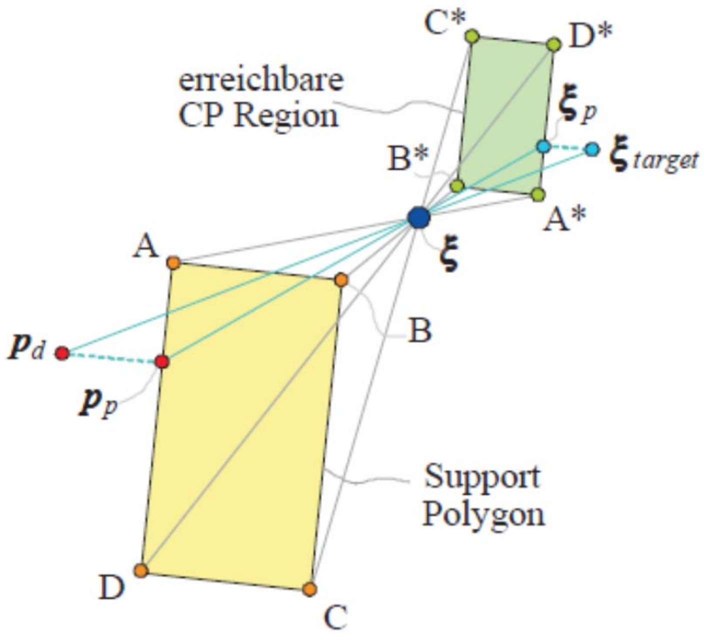




Capture Point Control

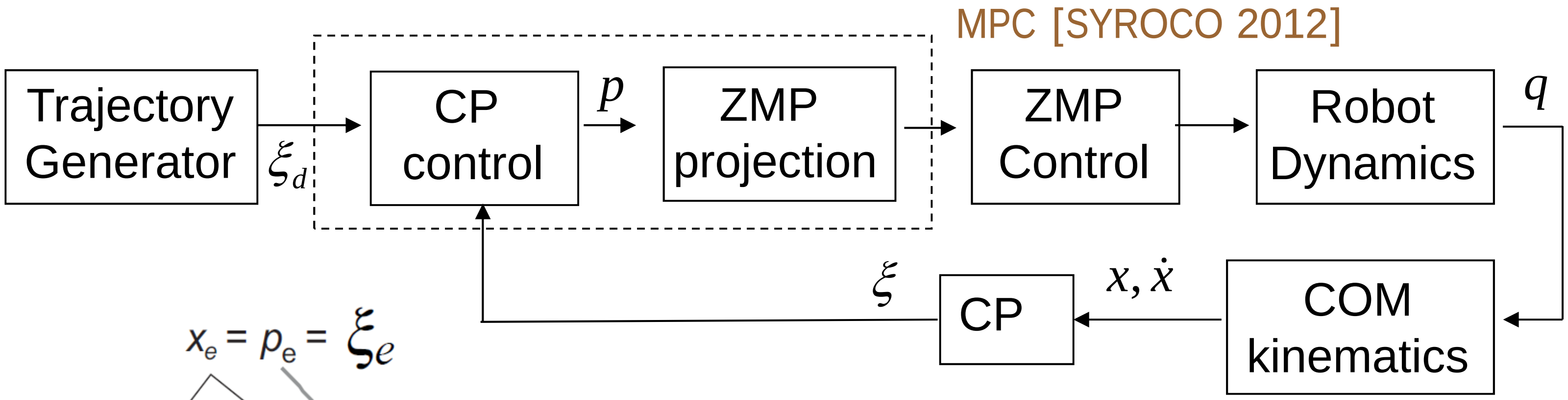


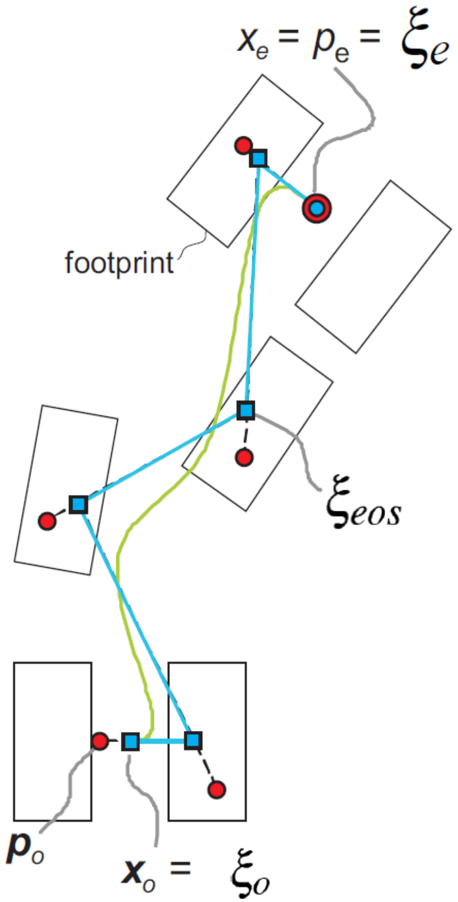


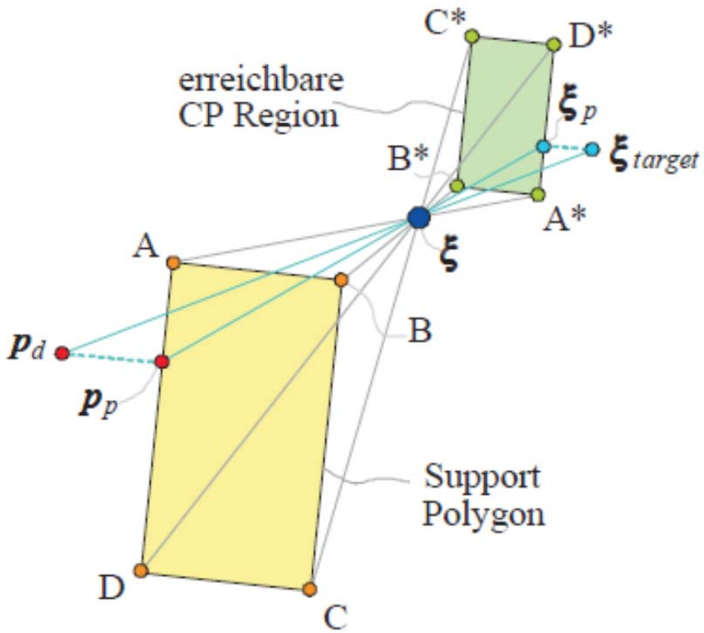


[Englsberger, Ott, et. al., IROS 2011]

Capture Point Control

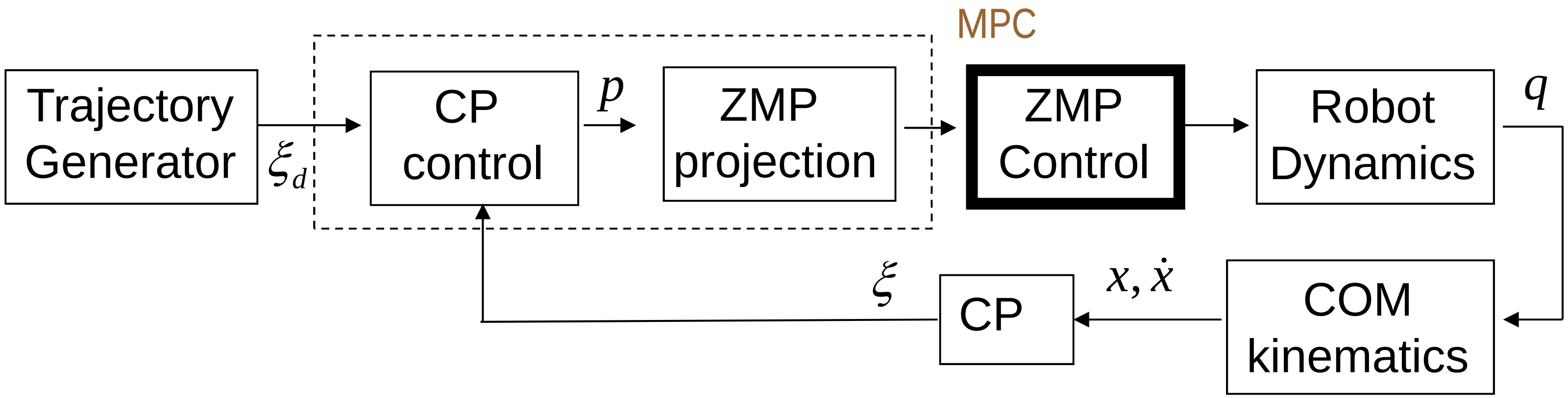






[Englsberger, Ott, et. al., IROS 2011]

Position based ZMP Control



Desired ZMP implies a desired force acting on the COM:

$$p_d \qquad \ddot{x} = \omega^2 (x - p) \qquad F_d = M\omega^2 (x - p_d)$$

Position based ZMP Control

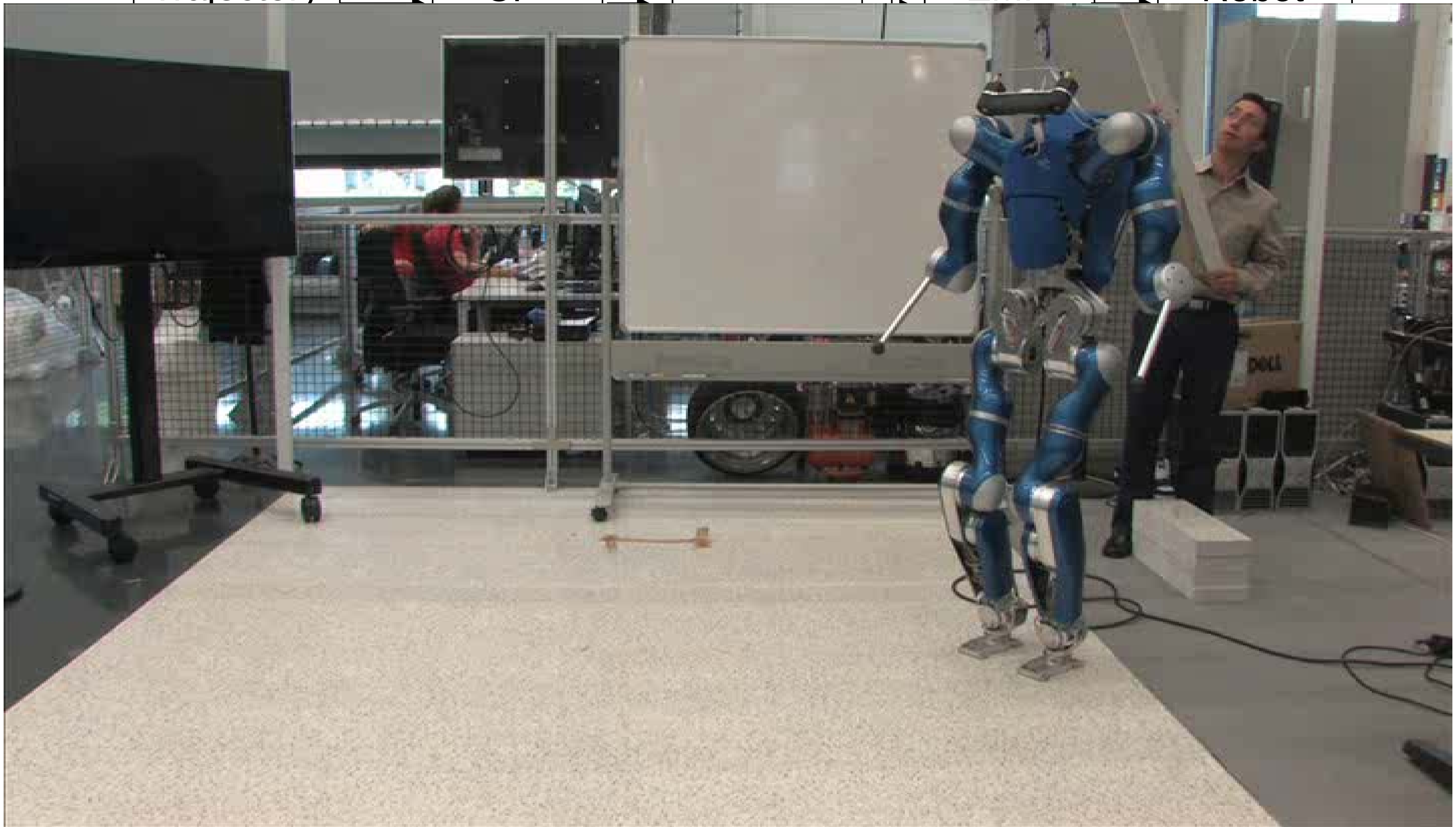
Position based force control [Roy&Whitcomb,2002]:

$$\dot{x}_d = k_f M\omega^2 (p - p_d) \qquad \dot{x}_d = k_f (F_d - F)$$

Capture Point Control

MPC [SYROCO 2012]

Trajectory | CP | p | ZMP | ZMP | Robot | q



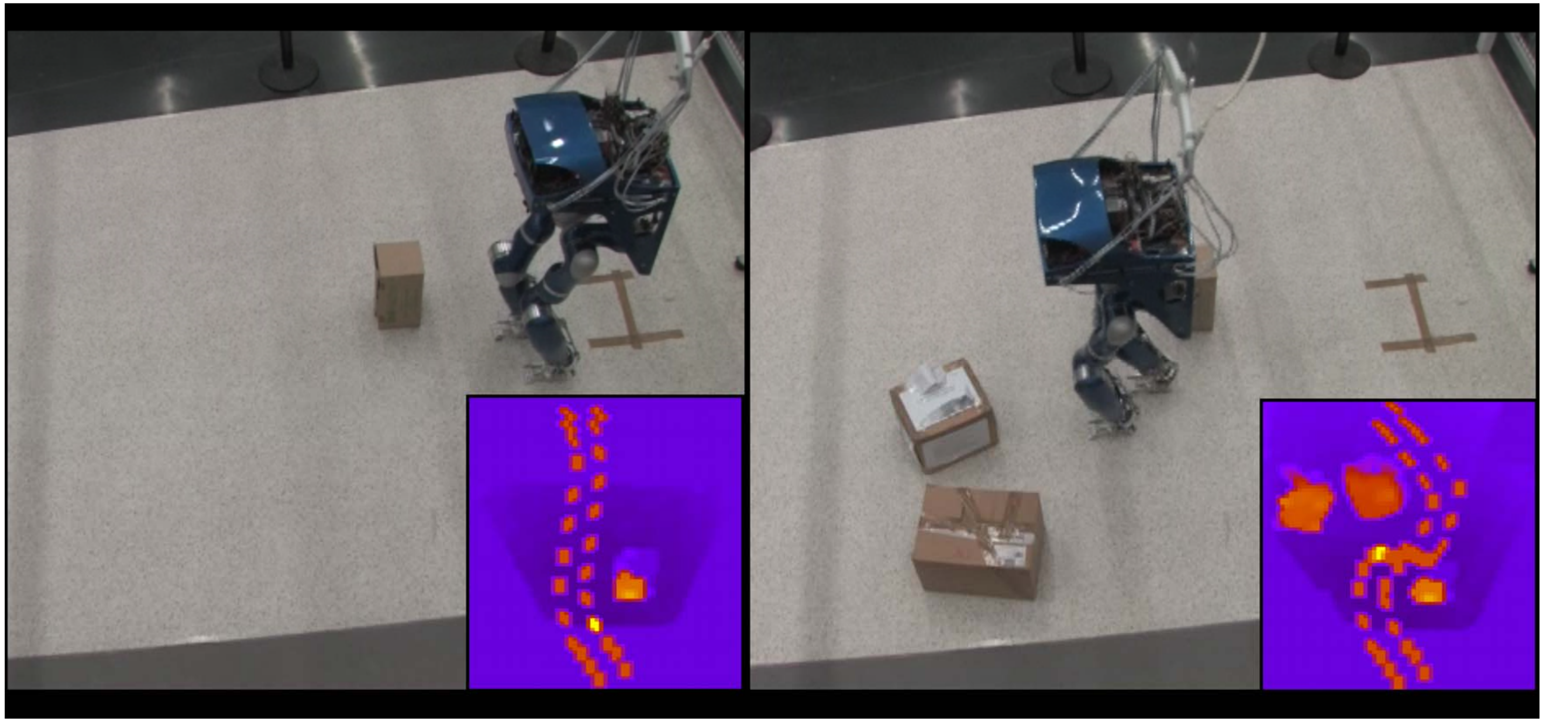
p_o $x_o = \xi_o$

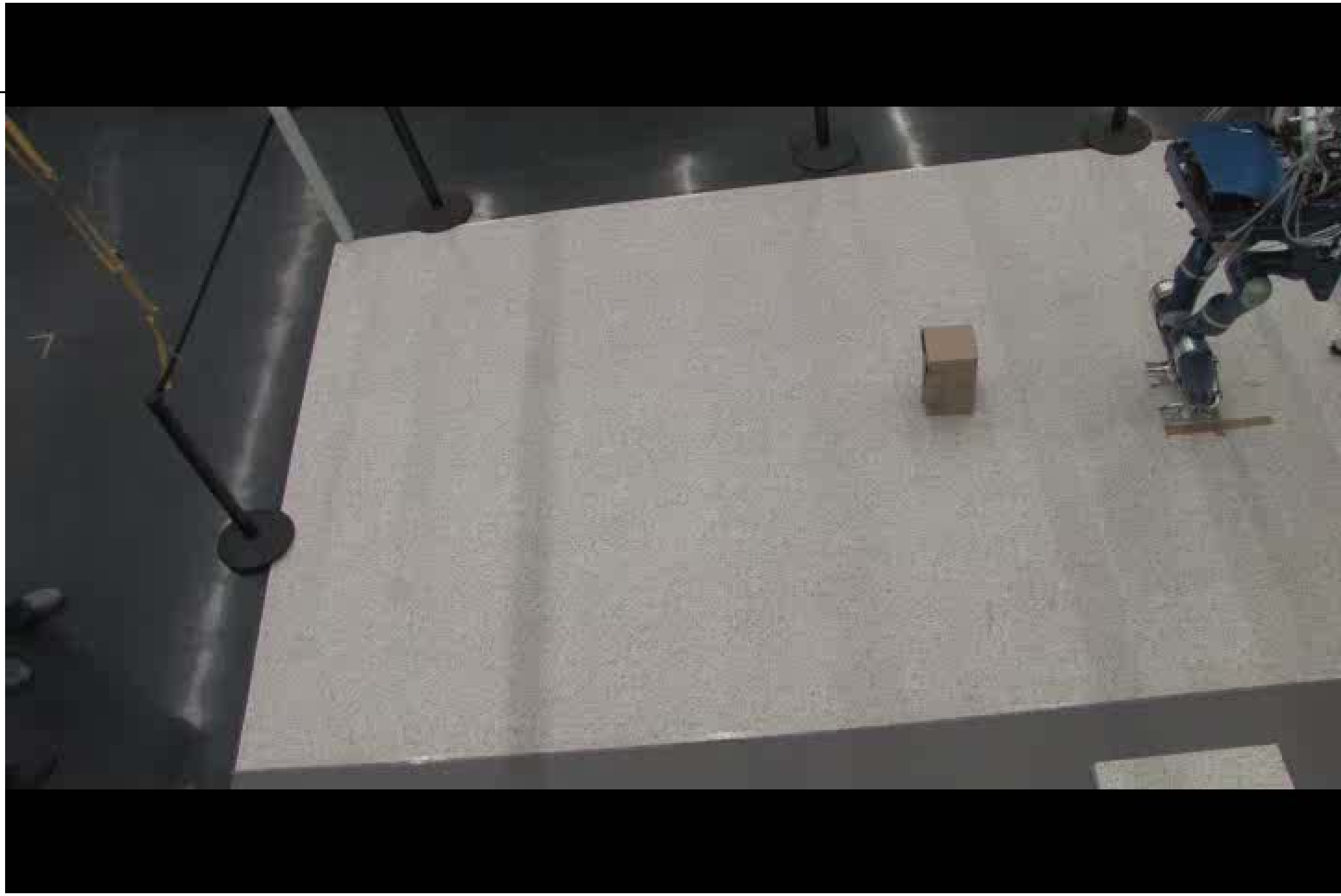
[Englsberger, Ott, et. al., IROS 2011]

Applications

1) Vision based walking

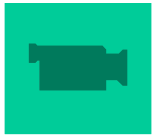
- stereo vision (Hirschmüller)
- visual SLAM (Chilian, Steidel)
- online footstep planning, collaboration with N. Perrin (IIT)





Applications

2) Optimized swingfoot trajectories: collaboration with H. Kaminaga (Univ. Tokyo)



- stride length: 70 cm
- speed: 0.5 m/s
- kinematically optimized swingfoot trajectory

- stride length: 70 cm
- speed: 0.5 m/s
- kinematically optimized torso motion
  (no angular momentum conversation! → slippery)

Summary

Part I:	Part II:	Part III:
Modeling	Balancing	Walking
Full Body Models →	Torque based Balancing	Pattern generation
Simplified Models →	ZMP based balancing →	Feedback control

Thank you very much for your attention!

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